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MAGNETICALLY LEVITATED SPACE ELEVATOR TO LOW-EARTH ORBIT*

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Abstract

The properties of currently available NbTi superconductor and carbon-fiber structural materials enable the possibility of constructing a magnetically levitated space elevator from the earth's surface up to an altitude of ≈200 km. The magnetic part of the elevator consists of a long loop of current-carrying NbTi, composed of one length that is attached to the earth's surface in an east-west direction and a levitated-arch portion. The critical current density of NbTi is sufficiently high that these conductors will stably levitate in the earth's magnetic field. The magnetic self-field from the loop increases the levitational force and for some geometries assists levitational stability. The 200-km maximum height of the levitated arch is limited by the allowable stresses of the structural material. The loop is cryogenically cooled with helium, and the system utilizes intermediate pumping and cooling stations along both the ground and the levitated portion of the loop, similar to other large terrestrial cryogenic systems. Mechanically suspended from the basic loop is an elevator structure, upon which mass can be moved between the earth's surface and the top of the loop by a linear electric motor or other mechanical or electrical means. At the top of the loop, vehicles may be accelerated to orbital velocity or higher by rocket motors, electromagnetic propulsion, or hybrid methods.

Key words: levitation

1 Introduction

The concept of creating a mechanical structure attached to the earth's surface, upon which one could transport materials and people to and from space by means of an elevator, has been known since at least the time of Tsiolkovsky in 1895 [1]. In 1960 Artsutanov, and later others independently, introduced the notion that a mechanical elevator cable, sometimes referred to as a "sky-hook", could be stably suspended about a geosynchronous orbital location, 36,000 km above the earth's equator [1]-[3]. The usual contemporary design concept for a space elevator is based around a mechanical cable extending radially inward and outward from a geosynchronous orbit, usually with a counterweight at the outer radius and with the innermost part of the cable attached to the ground at the equator. A design constraint of this system is that the yield strength can not be exceeded at any point on the cable. Unfortunately, currently available materials are not strong enough to support their own weight in a constant-cross-sectional-area cable from an earth geosynchronous location to the earth's surface. In principle, such a cable can be constructed by tapering the cross section from a small diameter at the ends to a very thick diameter at the geosynchronous point [2]. In practice, the strength of presently available engineering materials makes the mass of such a cable uncomfortably large.

In this paper, we present an alternative concept for a space elevator. The basic idea is to use the Lorentz force on a current-carrying cable in the earth's magnetic field to levitate into low-earth orbital altitudes the wire, its cryogenic coolant and mechanical sheath, together with a mechanical structure for an elevator that is suspended from the mechanical sheath. The properties of currently available NbTi superconductor allow this possibility. A recent paper discussed aspects of the cryogenics and cost [4]. This paper discusses the magnetic and structural aspects of the magnetically levitated elevator.

2 Basic Concept

To discuss the basic concept, consider a very simplified system, shown in Fig. 1, that consists of a current loop with current I, oriented perpendicular to an external uniform magnetic field B that is constant in time. The loop consists of four parts: (1) a segment of length L + 2R that is securely connected to the ground, (2) a quarter-circle arc of radius R, (3) a horizontally levitated segment of length L, located at altitude R above the ground, and (4) another quarter-circle arc of radius R. The direction of I and B are such that Segment 3 experiences a vertical upward force, Segment 1 experiences a vertical downward force, and Segments 2 and 4 experience a force radially outward along their entire length. For the moment, we ignore the self-field of the coil and many other first-order considerations that affect the detailed shape of the levitated segments and the viability of a real system.

Earnshaw's theorem teaches that a totally unconstrained magnetic system cannot be made statically stable [5]. However, while Segments 2, 3 and 4 are mechanically uncon

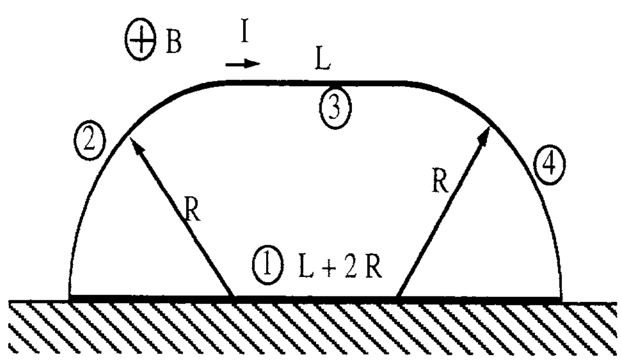

Figure 1. Schematic diagram of general concept, showing a magnetically levitated elevator, consisting of (1) a ground-based segment of length L + 2R, (2) and (4) arc segments of radius R, and (3) levitated horizontal segment of length L at altitude R.

strained, Segment 1 is attached to the ground, and the connections of 2 to 1 and 4 to 1 can be considered as a mechanical ball rotation joint. For some current I_0, the upward vertical force on segments 2-4 will exceed their weight, the assembly will levitate, and tension will be produced in the cable. For sufficient tension in the cable, the shape of the cable should be maintained. Further, in a constant horizontal magnetic field this levitation is statically stable against rotations of the loop from the ball joints.

The current loop will consist of an assembly of superconducting cables, cryogenic cooling, thermal insulation, and a mechanical support that accommodates the tension that arises in the cable. Mechanically suspended from the basic loop is an elevator structure, upon which mass can be moved between Segments 1 and 3 by a linear electric motor or other mechanical or electrical means. The entire assembly can be considered analogous to a suspension bridge, with the superconducting cable analogous to the main supporting cable, and the elevator structure analogous to the bridge deck. In this case however, rather than being supported at a few towers, the main cable is at least partially supported everywhere along its length by the magnetic levitation force.

Along Segment 3, vehicles may be accelerated to orbital velocity or higher by rocket motors, electromagnetic propulsion, or hybrid methods such as the maglifter concept [6]. Unlike such actions at the earth's surface, the accelerations will not be resisted by atmospheric drag. The acceleration of gravity along Segment 3 will be only slightly less than that of the surface, however, the pressure should be that of the best vacuums attainable on the earth's surface. As in the geosynchronous space elevator concept, the magnetically levitated elevator may enable a space transportation system that is less expensive, safer, and more environmentally friendly than systems based on chemical rockets alone.

3 Cable Levitation

3.1 Horizontal Segment

In the simplified problem shown in Fig. 1, Segment 3 is assumed horizontal. The levitational force is then independent of any horizontal tension in the segment. The condition for levitation from the earth's surface is that the Lorentz magnetic force F_m be greater than the gravitational weight of the conductor plus any superstructure it is attached to, i.e.,

$$B\,I\,L > m\,g\,,$$

where m is the mass of the cable and g is the acceleration of gravity. The current in the cable is the product of the current density j and the conductor area A. The mass is the product of density per unit volume ρ, A, and L. The minimum levitation condition is that in which the conductor levitates just itself,

$$B\,j > \rho\,g\,. \tag{1}$$

The horizontal component of the earth's magnetic field is ≈25–30 μT at the surface [7]. If we assume a density approximately equivalent to that of NbTi superconductor, i.e., $\rho = 9000\ \mathrm{kg/m^3}$, we have a minimum allowable current density of

$$j_{min} = (9000)(9.8)/(2.5 \times 10^{-5}) = 3.5 \times 10^9\ \mathrm{A/m^2}.$$

The state of the art in the fabrication of NbTi superconductor is that at a temperature of 4 K, this j may be realized in magnetic fields as high as 5 T [8]. For fields of only 2 T, current densities as high as $1.5 \times 10^{10}\ \mathrm{A/m^2}$ have been achieved [8] and should be sufficiently high to levitate the superconductor plus a significant amount of support infrastructure.

To first order, the earth's magnetic field can be approximated as that of a dipole, such that

$$B \propto 1/R_e^{\,3}\,, \tag{2}$$

where R_e is the distance from the center of the earth. The gravitational force is such that

$$g \propto 1/R_e^{\,2}\,. \tag{3}$$

The radial dependence of (2) and (3) are such that for fixed current I, satisfying the condition of (1), there is an altitude R at which the segment is vertically stable.

3.2 Arc Segment

Analysis of the forces along Segment 2 or 4 requires consideration of the cable tension T and uses the schematic shown in Fig. 2. Using parameter s as the distance along the cable from the origin, a force balance, with the vector $\mathbf{T_o}$ the supporting force at s = 0, requires

$$\mathbf{T_o} + \int_0^s \mathbf{f}(s)\, ds + \mathbf{T}(s) = 0\,, \tag{4}$$

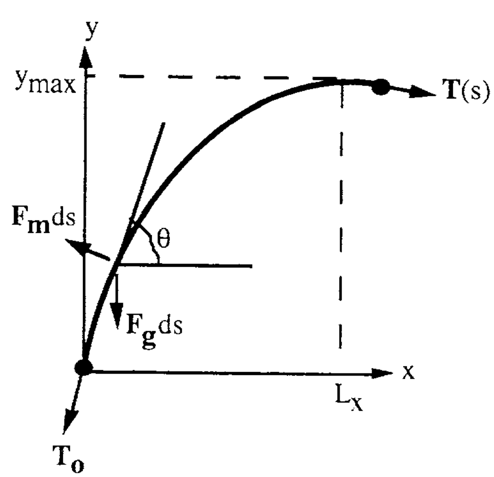

Figure 2. Force diagram for arc segment.

where **f** is the force per unit length and given by

$$\mathbf{f} = -F_g\,\mathbf{j} + F_m \cos\theta\,\mathbf{j} - F_m \sin\theta\,\mathbf{i}\,, \tag{5}$$

where **i** and **j** are unit vectors in the x and y directions, respectively, $F_g = \rho g A$ is the weight per unit length, θ is the angle the cable makes with the horizontal, and F_m is the magnetic force per unit length. Area A, and therefore F_g, may be a function of s. Dividing (4) into components and differentiating, we have

$$d/ds\,(T\cos\theta) = F_m \sin\theta\,, \tag{6}$$

$$d/ds\,(T\sin\theta) = F_g - F_m \cos\theta\,, \tag{7}$$

Multiplying (7) by cosθ, (8) by sinθ, and adding the results yields

$$dT/ds = F_g \sin\theta\,. \tag{8}$$

This result is valid regardless of the exact shape of the arc segment. If we assume that F_g is constant, we may integrate (8) along the arc, yielding

$$T(y) = T_o + F_g y\,, \tag{9}$$

and the condition for stability is that $T_o > 0$. This result shows that the cable tension at the top must support the weight of an equivalent cable with the same density that hangs straight down, plus any cable tension at the attachment point.

If we continue the assumption that F_m is uniform, then we may take the x component of (4) and integrate to a height y to obtain

$$T_x(y) = F_m\,y + T_{xo}\,, \tag{10}$$

where T_{xo} is the horizontal component of $\mathbf{T_o}$ and is assumed positive in the negative **i** direction. Comparing (9) and (10) at $y = y_{max}$,

$$T_o + F_g\,y_{max} = F_m\,y_{max} + T_{xo}\,, \tag{11}$$

and we come to the same relationship found for the horizontal part of the cable, i.e.,

$$F_m > F_g\,. \tag{12}$$

If the cable is vertical at the ground connection, then the amount of F_m that exceeds F_g will produce the tension at the ground connection, T_o.

We may integrate (7) to obtain

$$T_y(y) = F_g\,s - F_m\,x + T_{yo}\,, \tag{13}$$

where T_{yo} is the vertical component of $\mathbf{T_o}$ and is assumed positive downward. Letting $s = L$ at $x = L_x$, (13) yields

$$T_{yo} = F_m\,L_x - F_g\,L\,. \tag{14}$$

For levitational stability, $T_{yo} > 0$, which produces a more stringent constraint than (12), i.e.,

$$F_m\,L_x - F_g\,L > 0\,. \tag{15}$$

Substituting the values of T_{xo} in (11) and T_{yo} in (13) into the definition of T_o^2, we obtain

$$T_o = [(F_m L_x - F_g L)^2 + (F_g - F_m)^2 y_{max}^2]/$$
$$[2(F_m - F_g)y_{max}]. \tag{16}$$

With the assumptions made here, we can analytically solve for the cable shape of the arc. We start by solving for the total length L of the arc

$$L = \int_0^{y_{max}} \csc\theta\,dy\,, \tag{17}$$

which has for solution

$$L = [L_1 + L_2(\pi/2 + s_1)]/d_1\,, \tag{18}$$

where

$$L_1 = F_g\,y_{max}^{0.5}[2T_o - (F_m - F_g)\,y_{max}]^{0.5}\,,$$

$$L_2 = F_m\,(T_o + F_g y_{max}) / (F_m + F_g)^{0.5}\,,$$

$$s_1 = \sin^{-1}[(F_m y_{max} - T_o) / (F_g y_{max} + T_o)]\,,$$

$$d_1 = (F_m + F_g)\,(F_m - F_g)^{0.5}\,.$$

Because T_o is a function of L, (18) can be considered a transcendental equation for L with F_m, F_g, L_x, and y_{max} as parameters. Once L is known, we can solve for x as a function of y

$$x = \int_0^{y} \cot\theta\,dy\,, \tag{19}$$

which has for solution

$$x = [x_1 + x_2 + x_3\,(s_1 + s_2)]/d_2\,, \tag{20}$$

where

$$x_1 = F_m\{[y_{max}(F_m - F_g)]\,[2T_o - (F_m - F_g)y_{max}]\}^{0.5}$$

$$x_2 = -F_m[y_{max}(F_m - F_g)]^{0.5}[2T_o y_{max} -$$
$$(F_m - F_g)y_{max}^2 + 2(F_m y_{max} - T_o)y - (F_m + F_g)y^2]^{0.5}$$

$$x_3 = F_g(F_g y_{max} + T_o)y_{max}[(F_m - F_g)/(F_m + F_g)]^{0.5}$$

$$s_2 = \sin^{-1}\{[(F_m + F_g)y + T_o - F_m y_{max}]/(F_g y_{max} + T_o)\}\,,$$

$$d_2 = (F_m + F_g)\,(F_m - F_g)\,.$$

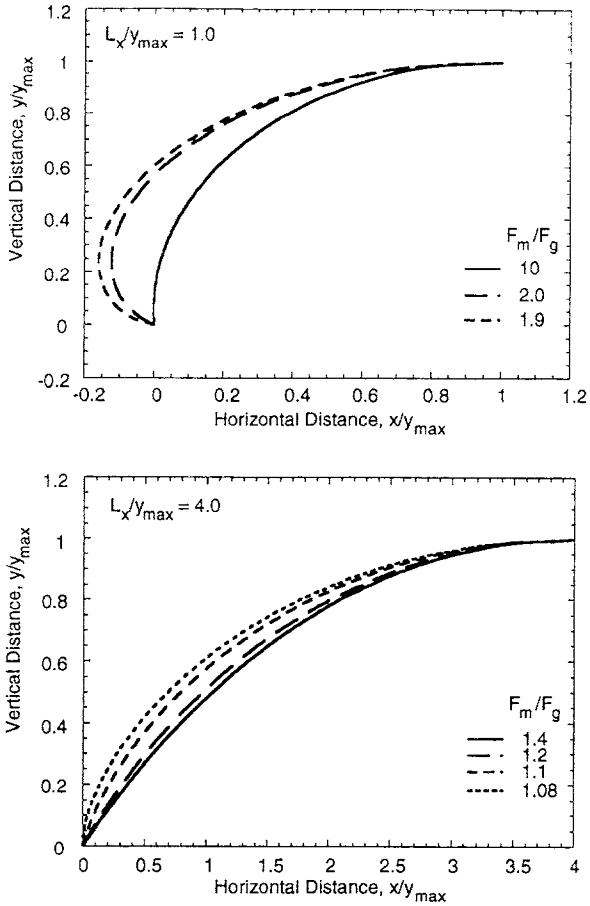

Figure 3. Profiles of arc segments of cable with L_x and F_m/F_g as parameters.

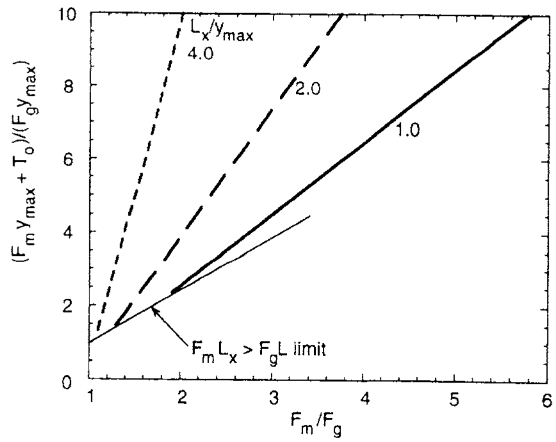

Figure 4. Normalized cable tension versus F_m/F_g for several L_x/y_{max}.

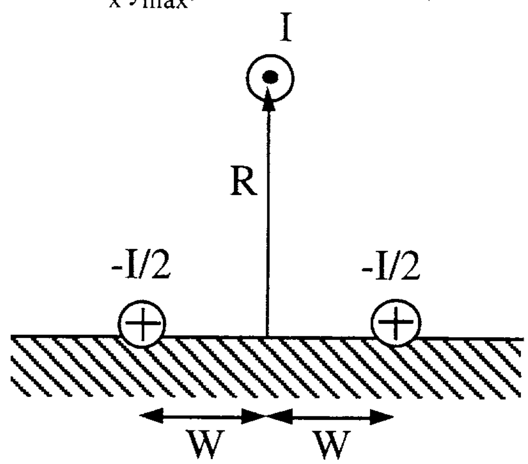

Figure 5. East-west cross section, showing current of ground-based segment split into two equal conductors, a horizontal distance W from the latitude of the levitated segment.

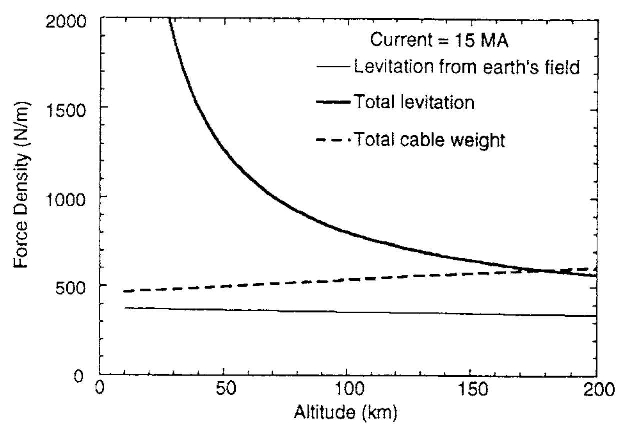

Fig. 6. Specific weight and levitational force as function of altitude for cable current of 15 MA.

Using (20), we show cable profiles in Fig. 3 for several values of L_x/y_{max} with F_g/F_m as a parameter. The solution procedure also yields the cable tension at the top of the arc, which is plotted in Fig. 4 as a function of F_m/F_g for several values of L_x/y_{max}.

From examination of Figs. 3 and 4, one can understand several properties of the levitated loop. Levitational stability establishes a lower limit on the allowed ratio of F_m/F_g, and this limit increases with decreasing L_x/y_{max}. For not unreasonably large L_x, one may approach the ratio of unity for this lower limit, and the maximum cable tension also approaches the limit $T_{max} = F_g y_{max}$, which was also noted as a result of (9). One is able to achieve this low cable tension only at the expense of significant amounts of horizontal extent of the arc portion of the loop. This penalty is partially mitigated by the shape of the arc segments, which have about half of their horizontal extent at an altitude above $0.8y_{max}$.

3.3 Self-Field Levitation

For sufficiently large currents in the loop, the self field of the loop may rival that of the earth's field. For a single filament conductor, the general result of the self field is to produce tension and increase the levitational force. However, the force on the levitated segments is always directed radially from the loop, and such a force is not stabilizing. The combination of self field and earth's field should be stabilizing for small departures from the equilibrium levitated position.

By establishing a horizontal separation of the ground-based segment of the loop, e.g., as shown in Fig. 5, a self field can be established that is stabilizing for the levitated portions. At the equilibrium position, the horizontal segment is vertically stable from the magnetic fields alone.

Movement in the horizontal direction always results in a radially outward force that will produce a stabilizing tension in the cable. Movement along an arc of radius R always produces a restoring torque. In addition, the earth's field will always act to increase the stability of the levitated segments. The ground-based horizontal segments must join together at the ball joints, where the quarter-circle arcs connect to the ground. Fig. 6 shows the relative contributions of self and earth's field to the levitation as a function of conductor height, based on a design example from [4]. While it is possible to power the cryogenic refrigerators disperse on the loop from the superconducting cable, in this example it was assumed that the power supplies for the refrigerators would be carried on the cable, thus increasing the parasitic weight.

3.4 Initial Levitation

For a single cable in the earth's field the cable may be laid on the ground and current increased until the cable begins to levitate. Current may be increased further to bring the free segments into their final levitated positions. The initial levitation of a loop that utilizes the self field (e.g., as in Fig. 5) for most of its levitational force is somewhat more complicated. Here, one would initially place the segment to be levitated on the ground in the middle of and slightly above the ground-based segments. The arc segments of the cable would need to be contained in some spooling arrangement. As current is increased in the loop, the levitated segment rises, and the spools let out some of the arc segments. The horizontal stability of the levitated segments during initial levitation is similar to that in the final location.

3.5 Off-Equator Operation

It may be inconvenient to build a space elevator at the magnetic equator. Here we consider construction at some latitude at which there is a vertical and horizontal compo

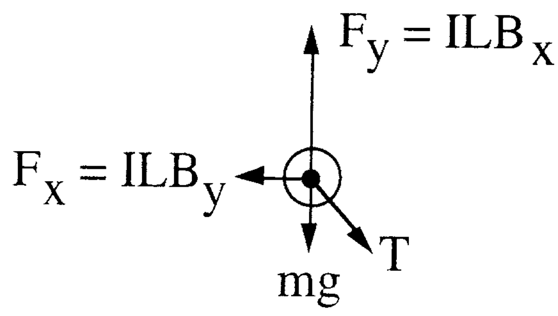

Figure 7. Force diagram for segment with non-horizontal magnetic field.

nent to the earth's field. If the self field is significantly larger than the earth's field, then a system analogous to that shown in Fig. 5 can be constructed. Further, the loop may be oriented in almost any horizontal direction.

If the earth's field dominates, then, as indicated in Fig. 7, there must be vertical and horizontal tension in the cable to obtain stability. This changes the shape of the cable and increases the maximum cable tension for a given maximum altitude. If the vertical component of the field is larger than the horizontal component, then it is impossible to levitate by the earth's field alone.

4 CONDUCTOR DESIGN

Larger diameter cables are useful for attaching larger mechanical structures to them. Smaller diameter cables are preferredfor minimizing the extraneous mass that needs to be levitated. The cable design is governed by the need to keep the self magnetic field reasonably low, which tends to disperse the current to larger radii. It is desirable to construct the cable in parallel components for structural, cryogenic, and electrical integrity from disruptions due to meteor strikes and other adverse events.

The local self field of the cable is much greater that the earth's field or that from the loop itself. Thus, the rela

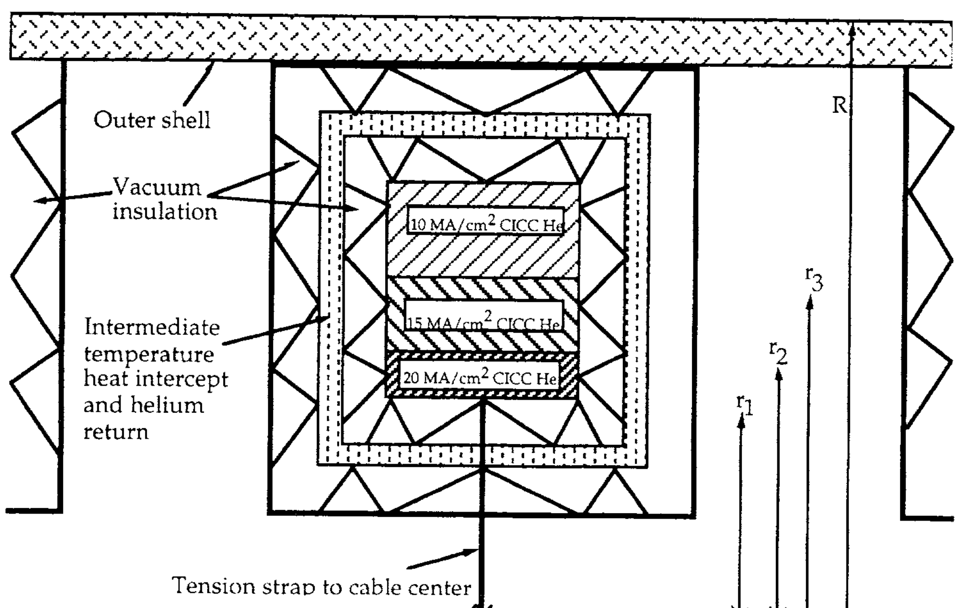

Figure 8. Cross section at one of the cable conductor elements.

tionship between magnetic field and radius r of the conductor from the cable centerline is well approximated by Ampere's law

$$B = \mu_0 I / (2\pi r) , \qquad (21)$$

where $\mu_0 = 4\pi x 10^{-7}$ T/Am is the magnetic permeability. Assuming that a magnetic field of 3 T is the maximum useful self field, then (21) yields that for r = 1 m, I = 15 MA.

The conductor mass can be reduced by taking advantage of the higher critical current densities at lower fields and placing these conductors closer to the center. For purposes of calculating conductor weight, we assume here that the superconductors are divided into three concentric shells of radius r_1, r_2, r_3, capable of sustaining magnetic fields of 1, 2, and 3 T, respectively. This yields estimated j_c of 20 GA/m^2, 15 GA/m^2, and 10 GA/m^2, respectively in each shell [8]. An optimal current distribution would put the three radii as close to the maximum as possible and result in approximately equal current in each shell. This arrangement and a possible geometry for the a cable element is shown in cross section in Fig. 8. For thermal stability of the superconductor, normal conductor is often fabricated in close proximity to the superconductor, and we assume a ratio of Cu:SC of 1.3. The major force on the conduit, caused by the local self field of the cable, is constrained by a radial tension strap. There are also some small amounts of mechanical constraint on the sides and above the conduit.

5 DISCUSSION

A comprehensive discussion of the many engineering problems associated with the magnetically levitated space elevator are beyond the scope of this introductory paper. However, some of the more obvious issues are briefly mentioned here. If for some reason the mass of levitated structure needs to be increased, it should be possible to install new superconducting cable by threading it along the existing levitated structure, or by levitating it independently from the ground. Design for repair in the field in pressure, etc. should be challenging. The environmental impact of the large local change of the earth's magnetic field due to the large magnetic structure will likely receive considerable attention. In a conventional sky-hook concept, the lower limit of size is determined by drag forces from atmospheric winds [3]. The platform of a levitated elevator could be reached by a sky-hook not subject to this constraint. The cable is designed such that if one segment of one strand becomes normal the current may be quickly shared by all of the other strands of the segment, and the cable should be able to stably levitate until superconductivity in the misbehaving strand segment returns. However, failure scenarios need to be developed for the unlikely event that the entire cable quenches.

Estimated construction cost of the magnetically levitated space elevator is well within the ability of many industrial nations [4]. A preliminary economic analysis estimates the cost to orbit at < \$30/kg when amortized over ten years with a large volume of traffic [4].

6 CONCLUSIONS

Presently available NbTi superconductor and structural materials enable the possibility of constructing a magnetically levitated space elevator from the earth's surface up to an altitude of approximately 200 km. Advantages of such a space elevator over those anchored at geosynchronous orbits are that it can be made from presently available materials, it can be employed initially from the earth's surface, it allows a long stretch of horizontal run at the top without penalty of additional tension, and it need not necessarily be located on the equator.

The brief calculations presented in this paper suggest that the magnetically levitated space elevator may be technically and economically feasible. While the potential of such a system is intriguing, there is no guarantee that the feasibility will withstand deeper scrutiny. In this short introduction of the concept, we have ignored many practical problems that must be solved before such a structure can become a reality. We have not considered conditions in the ionosphere and in space that might have deleterious effects upon any material employed; we have not estimated the probability of collision with meteoroids or man-made satellites; and we have examined only some of the obvious linear problems of stability. The engineering problems inherent in this system will be answered only by a program commensurate with some of the large contemporary projects.

Acknowledgment

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