DETERMINATION AND CORRECTION OF THE LINEAR LATTICE OF THE APS STORAGE RING*

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Abstract

The APS storage ring is a very complicated machine containing 400 quadrupoles and 280 sextupoles, each powered separately. The quadrupole calibration errors and orbit errors through the sextupoles are two main sources of linear optics distortion. The linear optics of the APS storage ring has been experimentally calibrated using an orbit response matrix analysis. The results obtained were used to correct the β-function beating around the ring. These corrections significantly improved the lifetime and injection efficiency for the low-emittance lattice. The energy acceptance of the machine was also increased from 1.8% to 2.5%. In this paper we present the results of the response matrix analysis and discuss the difficulties arising from the large size of the machine.

1 INTRODUCTION

From the beginning of the APS storage ring operation there was a substantial difference between the linear model and the real storage ring. We even have to use empirical correction factors when transforming the model into the real machine to get the betatron frequencies correct. This results in difficulties when tuning the machine to new lattice conditions, such as the low-emittance lattice. That is why we decided to develop a method for linear lattice calibration using orbit response matrices. There are several other problems that can be solved using the response matrix fit method:

- BPM gain calibration.
- Local linear coupling characterization and correction.
- β-function measurements around the ring.

The orbit response matrix is the change in the orbit at the BPMs as a function of changes in orbit correctors. The response matrix is defined by the linear lattice of the machine; therefore it can be used to calibrate the linear optics in a storage ring. Modern storage rings have a large number of correctors and precise BPMs, so measurement of the response matrix generates a very large array of precisely measured data.

The main idea of the analysis is to adjust the quadrupole gradients of a computer model of the storage ring until the model response matrix best fits the measured response matrix. The method was first suggested (to the authors’ knowledge) by Corbett, Lee, and Ziemann at SLAC [1]. A very careful analysis of the response matrix was done at the NSLS X-ray ring [2] and at the ALS [3]. There are a number of papers in the Particle Accelerator Conference Proceedings that describe similar model calibration techniques.

The problem of fitting the response matrix is solved in the following way. Let the response matrix \( M \) be a function of the vector of variables \( x \). Then we need to solve the equation

\[
M_{\text{measured}} - M_{\text{model}}(x) = 0,
\]

which can be solved by Newton’s method:

\[
\Delta x = \left( \frac{\partial M_{\text{model}}}{\partial x} \right)^{-1} \left( M_{\text{measured}} - M_{\text{model}}(x_0) \right),
\]

where \( x_0 \) corresponds to the initial model. To fit the response matrix, we have to determine all variables on which the response matrix depends, calculate the derivative of the response matrix with respect to these variables, and then invert it. After that, the solution can be found by iteration.

The most obvious and important variables are focusing errors (quadrupole calibration errors or orbit errors in sextupoles), corrector calibration errors, and BPM gain errors. Another obvious but less important set of variables is the energy shift associated with the changing of each corrector. These are the variables that are used for the response matrix fit described in this paper. The decision on what variables to use depends on details of the particular storage ring and how accurately the response matrix can be measured.

2 APPLICATION TO APS

2.1 Difficulties: model size and degeneracy

Up to this time, the most comprehensive analysis of the response matrix has been done at the NSLS X-ray ring and at the ALS. These two storage rings are smaller than the APS. In case of the APS, if one would try to use all correctors and BPMs, there would be 2,240 variables to vary and about 560,000 elements to fit. The size of the response matrix derivative would be 9 Gb and is much larger than the memory size of an average computer. In addition, the computation time would be many days.

There are two sources of model degeneracy the APS storage ring has that the other smaller rings lack. First, the smaller rings are able to store the beam without the sextupoles powered. This allows them to separate two kinds of gradient errors: quadrupole imperfections and orbit errors in sextupoles. Second, the average betatron phase advance between quadrupoles at APS is a rather
small value, 0.088, while for the NSLS X-ray ring it is 0.17, and for the ALS it is 0.28.

2.2 Choice of variables

In order to limit the required computer memory usage and the computation time, we have to limit the size of the measured response matrix. The size of the uncoupled response matrix is

\[ N_{\text{elements}} = N_{\text{corr}} \times N_{\text{BPM}} \times N_{\text{corr}} \times N_{\text{BPM}}, \]

where \( N_{\text{corr}} \) and \( N_{\text{BPM}} \) are the number of correctors and BPMs in the x and y planes. The number of variables is

\[ N_{\text{var}} = N_{\text{quads}} + 2N_{\text{corr}} + N_{\text{BPM}} + N_{\text{corr}} + N_{\text{BPM}}, \]

where the factor of 2 in front of \( N_{\text{corr}} \) comes from the energy variation.

The obvious way to decrease the response matrix size is to reduce the number of correctors and BPMs in measurements. However, to achieve a high-precision fit, we have to use as many BPMs as possible. This leaves only one available option — reducing the number of correctors.

The most obvious minimal set of correctors is one corrector per sector. Then, in the case of one corrector, nine BPMs, and nine quadrupoles per sector per plane, the size of the response matrix derivative is

\[ N_{\text{var}} \times N_{\text{elements}} = 1200 \times 28800. \]

For double-precision calculations, the size of the response matrix derivative is about 260 Mb. The size of the computer memory required to invert the matrix and then manipulate it is about 1.2 Gb in this case. This set of variables is usually used for our calculations.

2.3 Measurements and fitting

The fitting process, which is just solving equation (1), is done in iterations. All accelerator-related calculations are performed using the code \textit{elegant} \cite{4}. The output of the fitting application is a file of fitted variables in the format of the “parameter” file of the \textit{elegant}. This file is used to update the ideal \textit{elegant} model of the storage ring. Before the fitting, a typical rms difference between the ideal model and the measured orbit distortions is about 50 \( \mu \text{m} \). After the fit is done, the rms difference is decreased to the noise level of the BPMs, which is about 1 \( \mu \text{m} \). This updated model can then be used for all kinds of calculations in \textit{elegant}, including calculation of the \( \beta \)-function around the ring.

After this nearly perfect agreement between the model and measured data is achieved, we have to ask: does this agreement necessarily imply a good agreement between the fitted model and the real elements in the storage ring? Although the number of data points (28800) is much greater than the number of fit parameters (1200), this does not guarantee the solution is unique. The redundancy of quadrupoles in the storage ring and the lack of ability to store the electron beam without sextupoles make it impossible to determine separate quadrupole errors. In other words, it is very likely that the measured response matrix can be reproduced (within the accuracy of the measurements) using different sets of quadrupole gradients. However, one would expect that the other varying parameters like BPM gains and corrector calibrations should be unique.

The easiest way to confirm the above statement is to measure several orbit response matrices, analyze each one separately, and see how much variation there is between the fit parameters for the different data sets. Figure 1 shows the results of fitting of two different measurements using the two different configurations of correctors. As expected, the solution for the quadrupoles is ambiguous and shows considerable difference between the two sets. At the same time, the solution for BPM gains does not depend on the response matrix configuration.

![Figure 1. Some results of quadrupole (top) and BPM (bottom) calibrations using two different response matrices. As expected, different matrices resulted in different quadrupole calibrations but the same BPM gains.](image1)

![Figure 2. Relative difference in vertical \( \beta \)-functions calculated using two different response matrices. The rms difference is 1%.](image2)
As a confirmation that the two different quadrupole sets indeed represent the same response matrix, Figure 2 shows small differences between the vertical $\beta$-functions calculated by elegant for both quadrupole sets. In spite of the quadrupole calibration ambiguity, the fit provides a unique storage ring model in terms of $\beta$-functions. The relative rms difference between the $\beta$-functions calculated using two different sets is 0.5% for horizontal and 1% for vertical $\beta$-function. These numbers can be used as an estimation of the accuracy of the $\beta$-function determination.

2.4 Beta function beating correction

Since the solution for the quadrupole errors is ambiguous and varies from measurement to measurement, we cannot directly apply it with the opposite sign for the $\beta$-function corrections. Instead, the fitted model was used to calculate the $\beta$-functions, and then the SRbetaCorrection application was used to compute the quadrupole corrections. This application uses an inverse matrix multiplication to determine a set of gradient corrections. It was previously used to correct $\beta$-function modulation using a sparse set of measured $\beta$-functions from quadrupole scans [5]. The corrections were then applied to the storage ring, and the response matrix measurement and fit were performed again. The $\beta$-functions before and after correction are shown in Figure 3.

![Figure 3. Horizontal $\beta$-functions of the “low-emittance” lattice before (left) and after (right) correction.](image)

Corrected $\beta$-functions improve the symmetry of the machine; this in turn should improve the nonlinear beam dynamics. The positive effect of the correction was observed for the low-emittance lattice; the lifetime was increased by 40% and the injection efficiency was improved.

At the APS storage ring, the lifetime is defined by the nonlinear energy acceptance. To confirm that the energy acceptance was increased after the correction, the lifetime dependence on the rf voltage was measured. Figure 4 shows the lifetime vs. rf voltage taken on three different dates. The important feature of this plot is the gap voltage where the lifetime achieves a maximum (the overall lifetime is dependent on bunch pattern and coupling, which we did not reproduce for all measurements). This voltage is a measure of the energy acceptance. The first curve corresponds to the initial low-emittance lattice; the best lifetime is achieved at 8.0 MV. The second curve was measured after the first correction was applied; the maximum is achieved at 8.5 MV. This correction was based on the $\beta$-functions measured in several quadrupoles by the quadrupole scan method. The third curve was measured after the correction based on the response matrix fit. The lifetime maximum is achieved on or beyond 9.5 MV. Overall, the energy acceptance was increased from 1.8% to 2.5%.

We have to mention that not every attempt to correct the lattice was successful. This is due to the fact that the existing $\beta$-function correction application does not use all available quadrupoles. It also does not provide simultaneous correction of the dispersion. This shortcoming will be corrected in the future.

![Figure 4. Lifetime dependence on the rf voltage. The important feature of this plot is the gap voltage where the lifetime achieves its maximum.](image)

3 CONCLUSIONS

We have created precise linear models of the storage ring in terms of $\beta$-functions for both low-emittance and high-emittance lattices. Using these models, the $\beta$-function beating corrections have been successfully applied. The lifetime was increased by 40% for the low-emittance lattice as a result of the corrections. The models allow the user to apply predictable and precise changes to the existing lattice. For example, after applying the $\beta$-function corrections, the $\beta$-function changes exactly coincide with the changes predicted by the model.

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REFERENCES