AN INVESTIGATION OF AN OPTIMAL RANGE OF SUBCRITICALITY FOR ACCELERATOR-DRIVEN SYSTEMS

Y. Kim, W. S. Park,
Korea Atomic Energy Research Institute
150 Deokjin-dong, Yuseong-gu, Daejon 305-353, Republic of Korea
ykim@kaeri.re.kr; wonpark@kaeri.re.kr

W. S. Yang, T. A. Taiwo, and R. N. Hill
Argonne National Laboratory
9700 S. Cass Avenue Argonne, Illinois 60439-4842, USA
wyang@ra.anl.gov; taiwo@anl.gov; bobhill@anl.gov

ABSTRACT

It is attempted in this paper to define an optimal range of subcriticality of ADS systems from the operational and safety points of view. To devise a representative measure of the subcriticality level, the mathematical and physical implications of the effective multiplication factor and the source multiplication factor have been reviewed. A set of criteria that bound the feasible subcriticality level is proposed in terms of the effective multiplication factor; the minimum required subcriticality is determined by the largest value of potential reactivity increase including the temperature defect and the calculation and measurement uncertainties, and the maximum allowable subcriticality level is bounded by the system economy and the technical feasibility of the system. Within this feasible domain of subcriticality, a preliminary estimation of the optimal range of subcriticality was performed for a lead-bismuth-eutectic (LBE) cooled ADS design based on the safety and transmutation performances. The effects on the system safety of the subcriticality level were analyzed for several important transients using an integral safety analysis method, and the transmutation performance was evaluated in terms of the fuel and long-lived fission product discharge burnups.

1. INTRODUCTION

Various accelerator-driven systems (ADSs) have been proposed for transmuting the long-term hazard radioactive materials such as transuranic elements (TRUs) and long-lived fission products (LLFPs).[1,2] A major motivation of accelerator-driven system (ADS) was its potentially enhanced safety characteristics due to its subcriticality. However, it is not clear what level of subcriticality is enough to ensure the system safety. Furthermore, the optimal range of subcriticality has not been studied systematically.

Previous studies show that the subcriticality level is spread over a relatively wide range and the value is usually determined by a rule of thumb. During the early development stage of the ADS, it was considered that a higher subcriticality level would provide a better safety performance. However, it has been found that the higher subcriticality level can negatively impact the safety of ADSs in some situations.[3,4] This implies that there might exist an optimum range of subcriticality from the safety point of view. In addition, since the fuel inventory is determined by the subcriticality level, the fuel discharge burnup that is inversely proportional to the inventory is also affected by the subcriticality
level. In other words, the TRU transmutation performance that is the main function of proposed ADS systems is also dependent on the subcriticality level. This also implies that an optimal range of subcriticality may exist from the system performance point of view.

Therefore, it was attempted to estimate an optimal range of subcriticality of ADS systems from the operational and safety points of view. First of all, in order to determine the feasible domain of subcriticality, the safety requirements and the economical and technical constraints were examined. The minimum required subcriticality level was determined in such a way that the system does not become critical even with the largest value of potential reactivity increase including the temperature defect and the calculation and measurement uncertainties. On the other hand, the system economy and the technical feasibility of the system. Specifically, based on the result of a previous analysis for a typical ADS[5], the minimum multiplication factor required for an economic operation was employed as the economic constraint. As the technical constraint, the maximum beam power density that the beam window can hold out for a desired period was used since the beam power required to produce a desired fission power increases as the subcriticality level increases.

Within this feasible domain of subcriticality, a preliminary estimation of the optimal range of subcriticality was performed for an LBE-cooled ADS design based on the safety and transmutation performances. The effects of the subcriticality level on the system safety were analyzed for several important transients using a simple asymptotic analysis method, the so-called balance of power method[3]. In order to estimate the effects on the transmutation performance, the TRU and LLFP discharge burnups were evaluated by performing equilibrium cycle analyses using the REBUS-3 fuel cycle analysis code[6].

The objective of this paper is to provide a generic guideline for determining the optimum range of subcriticality level. In Section 2, the subcriticality measures for ADS are reviewed. The lower and upper bounds of the subcriticality level are derived in Section 3. Section 4 presents the effects on the safety of the subcriticality level for an LBE-cooled ADS design, and Section 5 shows the effects on the transmutation performance. Finally, conclusions are drawn in Section 6.

2. SUBCRITICALITY MEASURES FOR ADS

The degree of subcriticality of ADS can be represented in various ways.[7,8] However, the effective multiplication factor $k_{eff}$ and the source multiplication factor $k_{src}$ are most frequently used to represent it. Both of them are defined as the ratio of neutron production rate to loss rate, but $k_{src}$ is evaluated with the flux solution of the inhomogeneous source problem while $k_{eff}$ is evaluated with the flux solution of the homogeneous eigenvalue problem.

The flux distribution $\phi_s$ of a subcritical system with an external source $S$ is determined by the inhomogeneous equation

$$A\phi_s = F\phi_s + S,$$

where $A$ and $F$ are neutron loss and production operators, respectively. The source multiplication factor is defined with this source-driven neutron flux as

$$k_{src} = \frac{<F\phi_s>}{<A\phi_s>} = \frac{<F\phi_s>}{<F\phi_s> + <S>},$$

(2)
where \( < \cdot > \) denotes the integration over space and energy. On the other hand, the effective multiplication factor is defined as the eigenvalue of the following homogeneous equation (i.e., a system without the external source made artificially critical by use of an eigenvalue to scale neutron production):

\[
A\phi = \frac{1}{k_{\text{eff}}} F\phi .
\] (3)

By the definition of \( k_{\text{src}} \), \( k_{\text{src}}/(1-k_{\text{src}}) \) represents the number of fission neutrons produced per external source. Thus, the fission power level of an ADS for a given source distribution is determined by the source multiplication factor. This implies that the operational parameters such as the required accelerator power to maintain the desired power level and the burnup reactivity loss should be estimated using the source multiplication factor. In other words, the source multiplication factor indicates the degree of subcriticality from the operational point of view.

However, the source multiplication factor depends on the external source distribution, and hence it is not a characteristic of the ADS blanket itself. This causes some inconveniences in comparative design studies at the early design stage when the target design is not fixed. Furthermore, the inhomogeneous source problem has a steady-state solution only when the system is subcritical. If the system is no longer subcritical due to an accidental increase of reactivity, a simple time-independent source multiplication factor cannot be defined. The flux distribution increases as a linear function of time when the system reaches the critical state, and it increases exponentially when the system becomes supercritical. In the latter case, the exponential growth rate is determined by the degree of off-criticality. This suggests that from the safety point of view, the effective multiplication factor indicating the degree of off-criticality is a better measure of subcriticality level than the source multiplication factor.

The effective multiplication factor is independent of the external source, and hence it represents a system characteristic. In addition, it is generally easier to calculate than the source multiplication factor. Therefore, it is more convenient to use at the early stage of design studies. Although the effective multiplication factor is not directly related to the power level of a subcritical system, for a given source distribution, it can be related to the fission power level through the so-called source efficiency.[7] The source efficiency \( \phi^* \) is defined with \( k_{\text{eff}} \) and \( k_{\text{src}} \) as

\[
\phi^* = \frac{k_{\text{src}}}{1-k_{\text{src}}} / \frac{k_{\text{eff}}}{1-k_{\text{eff}}}. \] (4)

It indicates the efficiency of the source multiplication relative the multiplication of the fission neutrons. The source efficiency highly depends on the spatial and energy distribution of external source and the blanket configuration. However, for a fixed source distribution, it is relatively insensitive to the blanket design change. As a result, a scoping evaluation of operational parameters can be performed conveniently using the effective multiplication factor and the source efficiency.

Recalling that \( k_{\text{src}}/(1-k_{\text{src}}) \) is the number of fission neutrons produced by a single source neutron, the fission power \( P_{\text{fiss}} \) can be determined as

\[
P_{\text{fiss}} = \frac{E_f \phi^*}{\nu} \frac{k_{\text{eff}}}{1-k_{\text{eff}}} < S > ,
\] (5)
where \( E_f \) is the energy release per fission and \( \nu \) is the average number of neutrons produced per fission. Therefore, denoting the number of spallation neutrons produced by a proton by \( n_{sp} \), the proton current \( I_p \) required to generate a fission power \( P_{fiss} \) can be estimated by

\[
I_p = \frac{e_p \nu}{\varphi n_{sp}} \left( 1 - \frac{k_{eff}}{k_{eff}} \right) \frac{P_{fiss}}{E_f},
\]

where \( e_p \) is the electrical charge of proton.

### 3. FEASIBLE DOMAIN OF SUBCRITICALITY

#### 3.1 GENERAL CRITERIA

In the ADS design, a fundamental premise is that the core should be subcritical in both reloading and operational regimes, without any active intervention of the operator, e.g., control rod insertion. If the core reaches a critical state, the surmised advantages in terms of the safety could be seriously hampered and the motivation for such a core design can hardly be justified. Therefore, the subcriticality level of an ADS core should be sufficiently large enough to preclude the potential for criticality as a result of operational and accidental reactivity insertions.

Generally, an ADS core has a negative power defect, and hence the power reduction from the hot full power condition to the hot zero power state results in a positive reactivity insertion. Further temperature decrease from the hot zero power to the reloading stage also increases the core reactivity. Thus, the subcriticality level at the full power state should be sufficiently large enough to compensate for the reactivity increase (\( \partial k_{eff}^{TD} \)) resulting from the temperature swing from the hot full power state to a cold reloading state. In addition to this operational requirement for the subcriticality, it is also required that the core should remain still subcritical even in the case of an accidental insertion of a positive reactivity (\( \partial k_{eff}^{AC} \)). Furthermore, the uncertainties of the calculated \( k_{eff} \) and the value inferred from a measurement should also be considered in determining the upper limit of \( k_{eff} \).

On the other hand, the subcriticality level should be low enough to limit the accelerator power within the range determined by economical and technical constraints. As mentioned above, a previous study showed that \( k_{eff} \) should be greater than 0.95 for an economic operation.[5] In addition, technical feasibility related to the accelerator design and the coupling of accelerator and subcritical multiplier also imposes constraints on the lower bound of \( k_{eff} \) value since the beam power required to produce a desired fission power increases as \( k_{eff} \) decreases. These technical constraints depend on the design specifications such as the power level, external source configuration, etc. For example, a very high power system may require an accelerator current that is unavailable even with an advanced accelerator technology. Especially, if a beam window concept is utilized to couple the accelerator and the subcritical multiplier, the window integrity requirement may set a more stringent limit to the permissible accelerator current.

Based on these considerations, the feasible range of \( k_{eff} \) can be represented in a conservative way as

\[
\max(k_{eff}^{E\min}, k_{eff}^{T\min}) < k_{eff} < 1 - \partial k_{eff}^{TD} - \partial k_{eff}^{AC} - \partial k_{eff}^{UC} - \partial k_{eff}^{MU},
\]

(7)
where \( k_{\text{eff}}^{E_{\text{min}}} \) and \( k_{\text{eff}}^{T_{\text{min}}} \) denote the economical and technical lower bounds of \( k_{\text{eff}} \), respectively, and \( \delta k_{\text{eff}}^{\text{UM}} \) and \( \delta k_{\text{eff}}^{\text{UC}} \) indicate the uncertainties associated with the reactivity measurement and calculations, respectively. Since \( k_{\text{eff}} \) varies over a burn cycle, the lower and upper bounds of Eq. (7) should be applied to the smallest and the largest value of \( k_{\text{eff}} \) over a burn cycle, respectively. For example, in the case of TRU transmuter, the \( k_{\text{eff}} \) value decreases almost linearly over a burn cycle if no reactivity regulating device is employed. Therefore, the lower bound should be imposed on the \( k_{\text{eff}} \) value at the end of cycle (EOC), and the upper bound should be applied to the value at the beginning of cycle (BOC). As a result, denoting the burnup reactivity swing over a burn cycle by \( \delta k_{\text{eff}}^{\text{BU}} \), the BOC \( k_{\text{eff}} \) is bounded as

\[
\max(k_{\text{eff}}^{E_{\text{min}}}, k_{\text{eff}}^{T_{\text{min}}}) + \delta k_{\text{eff}}^{\text{BU}} < k_{\text{eff}} < 1 - \delta k_{\text{eff}}^{\text{TD}} - \delta k_{\text{eff}}^{\text{AC}} - \delta k_{\text{eff}}^{\text{UM}} - \delta k_{\text{eff}}^{\text{UC}},
\]

For a given system, the operational reactivity insertion can be readily evaluated. However, the potential reactivity insertion by accident is very difficult to determine since it depends on the accidents considered. If all the hypothetical accidents including the beyond-design-basis accidents are considered, the upper bound \( k_{\text{eff}} \) would be lower than the lower bound constrained by the economics and technical requirements. In this case, there is no feasible range of \( k_{\text{eff}} \) that satisfies all the constraints. In a fast spectrum ADS loaded with TRU or minor actinide (MA) fuel, the coolant voiding in the core generally induces a large amount of positive reactivity insertion. Furthermore, the fuel mass in a TRU (or MA) transmuter amounts to many times of the critical mass,[9] and thus a severe core disruptive accident involving a core compaction can lead to a huge positive reactivity insertion, which cannot be practically compensated for by the initial subcriticality. Therefore, for a practical and realizable ADS design, a compromise needs to be made for the determination of \( \delta k_{\text{eff}}^{\text{AC}} \), depending on the design features.

Based on the assumption that the probability for a hypothetical accident incurring a large reactivity insertion to occur is negligibly low in the ADS design, we propose to consider the following accidents in evaluating the quantity \( \delta k_{\text{eff}}^{\text{AC}} \) in Eq. (7). In the typical liquid target ADS design, the beam window is relatively vulnerable equipment whose failure could result in positive reactivity addition since the beam tube may be filled with the target material. Also, if control rods are employed to regulate the burnup reactivity loss, an inadvertent ejection of the rod should be considered. These two accidents may be considered as basic cases for determining the minimum required subcriticality of an ADS.

3.2 APPLICATION TO HYPER SYSTEM

All the parameters involved in the subcriticality range Eqs. (7) and (8) are dependent on the specific design characteristics, and hence a generally applicable feasible domain cannot be determined. Thus, a specific feasible domain was estimated using an ADS system named HYPER (Hybrid Power Extraction Reactor)[10,11], which is under development at the Korea Atomic Energy Research Institute (KAERI). Basic design features and core characteristics of the HYPER system are provided in the Appendix.

The operational reactivity change due to the temperature defect can be estimated using the temperature changes and the basic reactivity coefficients given in Table A.II.[12] In the HYPER
design, the coolant inlet temperature ($T_{in}$) and the coolant outlet temperature at the full power ($T_{out}$) are 340 °C and 510 °C, respectively, and the average fuel temperature ($T_f$) is 600 °C. Thus, assuming that the fuel is reloaded at a coolant temperature of 240 °C ($T_{load}$), the reactivity increase ($\delta \rho_{TD}$) from a full power condition to a cold reloading state was estimated as:

$$\delta \rho_{TD} = -(\alpha_T + \alpha_E)(T_f - T_{load}) - \alpha_{LBE} \left[ \frac{1}{2}(T_{in} + T_{out}) - T_{load} \right] - \alpha_R (T_{out} - T_{load}) = 454 \text{ pcm}$$

Limiting the accident cases to the window failure and the control rod ejection as discussed above, the accidental reactivity insertion ($\delta \rho_{AC}$) was estimated for the beam window failure since no control rod is utilized for the reactivity control. Under the assumption that beam tube is filled with the LBE target in the case of beam window failure, the reactivity increase due to the beam window failure was estimated to be 753 pcm as shown in Table A.II.

Based on these results, the maximum allowable value of $k_{eff}$ was estimated to be 0.988 without including the uncertainty terms. For a more practical estimation, the measurement uncertainties and the calculation uncertainties due to computational method errors and nuclear data uncertainties should be subtracted from this value. A recent experimental research reported that the reactivity of an experimental subcritical core could be measured accurately, within a few percent error, up to a subcriticality level of $k_{eff}=0.9$.[7] However, this measurement error is expected to be larger in an actual ADS system. Furthermore, because of large cross section uncertainties of MA isotopes, the calculated $k_{eff}$ uncertainties of ADS systems loaded with TRU or MA fuels would be much higher than those of the conventional fast reactors.

In the HYPER design, a single proton beam was adopted to simplify the core design, and a beam window is used to separate the proton beam delivery tube from an LBE target. As a result, the allowable beam current to ensure the window integrity is rather stringent. A preliminary evaluation showed that the maximum allowable proton current should be smaller than 20mA.[13,14] Therefore, the $k_{eff}$ value should not be smaller than the value required to produce the fission power of 1000 MWth with a beam power of 20 MW. This technical lower bound of $k_{eff}$ was approximately estimated by using Eq. (6). Using the estimated source efficiency (1.05) and the number of spallation neutrons produced by a proton (28) and assuming that $v = 2.9$ and $E_f = 200 \text{ MeV}$, the minimum required $k_{eff}$ was estimated to be about 0.961.

As shown in Table A.I, the $k_{eff}$ value of the current HYPER design at EOC is 0.951, and hence the current HYPER design cannot satisfy the technical boundary condition imposed by the beam window integrity. This requires further design modifications to reduce the burnup reactivity swing or to reduce the beam current density. Since the current design is optimized to minimize the burnup reactivity swing, however, it is difficult to reduce the burnup reactivity swing further without reducing the cycle length or the power level. In order to reduce the beam current density for a fixed beam current, the beam tube diameter needs to be increased, but this deteriorates neutronic performances and requires more shielding because of increased neutron leakage through the beam tube. Therefore, some compromise needs to be made among various design parameters including the power level, burnup reactivity swing, transmutation rates, and so on.
4. IMPACTS OF SUBCRITICALITY LEVEL ON SAFETY

4.1 BALANCE OF POWER METHOD

In order to evaluate the system dynamic behavior and safety characteristics, explicit dynamic and safety analyses need to be performed. However, since the relative effects of the initial subcriticality level on the accident consequences could be estimated by the asymptotic behavior, several important accident cases were analyzed using an integral safety analysis method. This method determines the asymptotic state after initial transient phase based on the balance of reactivity (BOR). It was initially developed for critical fast reactors in the late 80’s, and was used in analyzing the passive safety features of the Integral Fast Reactor (IFR).[12] This BOR method was recently modified into the balance of power (BOP) method for ADS system analyses.[3]

In the BOR method, the balance of reactivity (ρ) for critical reactors is represented as:[12]

\[ \rho = (P - 1)A + (P/F - 1)B + \delta T_{in} C + \delta \rho_{ext} = 0 \],

(9)

where \( P \) and \( F \) are normalized power and flow rate, respectively, and

\( (A + B) \) = reactivity coefficient experienced in going to full power and flow from zero power

isothermal at coolant inlet temperature,

\( B \) = power/flow reactivity coefficient,

\( C \) = inlet temperature reactivity coefficient,

\( \delta T_{in} \) = inlet temperature change from normal value \( T_{in} \),

\( \delta \rho_{ext} \) = external reactivity insertion.

This can be extended to ADS systems by representing the normalized power in terms of the initial subcriticality level and external source strength. Consider a subcritical system with an initial multiplication factor \( k_0 \) and an external source \( S \). If the system is perturbed by a reactivity addition of \( \delta \rho \) and an external source change of \( \delta S \), the asymptotic fission power at a perturbed state can be determined with Eq. (5) as

\[ P_{fiss} = \frac{E_f \phi^*}{\nu} \frac{<S> + <\delta S>}{(-\rho_0 + \rho)}, \]

(10)

where \( \rho_0 = 1 - 1/k_0 \). Assuming \( E_f \phi^*/\nu \) is constant, the normalized power \( P \) (normalized to unity at the initial state) can be written as

\[ P = \frac{\rho_0}{\rho} \frac{<S> + <\delta S>}{(\rho_0 + \rho)}. \]

(11)

Rearranging Eq. (11) for \( \rho \) and inserting into Eq. (9), the BOP equation for an ADS is obtained as

\[ P[\rho_0 + (P - 1)A + (P/F - 1)B + \delta T_{in} C + \delta \rho_{ext}] - \rho_0(1 + \frac{i}{i}) = 0, \]

(12)

where \( <\delta S>/<S> \) was replaced by \( \delta i/i \), the fractional change of the accelerator current \( i \). This equation is converted to the original BOR equation for a critical reactor if \( \rho_0 = 0 \).

The asymptotic power level at the perturbed state can be obtained by solving Eq. (12) for \( P \) as
Using this asymptotic power, the asymptotic outlet temperature can be determined with the coolant outlet temperature change \( \delta\text{out} \) defined as
\[
\text{cinout} - \text{out} + T - \Delta = \frac{1}{2\rho_0} (A + B) \frac{1}{F} \frac{1}{1 + \frac{\delta}{\rho_0}} (A + B - \rho_0 - \delta T_{in} C - \delta \rho_{ext} - \sqrt{\left[(A + B - \rho_0 - \delta T_{in} C - \delta \rho_{ext})^2 + 4\rho_0 (A + \frac{B}{F})(1 + \frac{\delta}{\rho_0})\right]}}{2(A + B / F)} \tag{13}
\]

where \( \Delta T_c \) is the coolant temperature rise at the full power/flow condition. It is important to note that the overall heat balance of the whole system including the secondary system is not modeled in the BOP method. In other words, the BOP method is based on the assumption that there is no intervention in the secondary system. In general, a change of the core outlet temperature results in a change of the inlet one with a significant time lag, which is usually several tens seconds depending on the system design. On the other hand, the core approaches quite fast to an asymptotic state in most transients, compared to this time lag. Thus, it can be a reasonable assumption that the inlet temperature does not change during a time period shorter than the time lag. For a realistic long-term behavior, a dynamic heat balance equation should be solved.

The validity of Eq. (12) is limited by the approximation \( E f / \nu = \text{constant} \). Although \( E f / \nu \) can be assumed to be a constant without introducing a significant error, the source efficiency \( \phi * ) may change during a transient, depending on the type of transient. The source efficiency generally depends on the geometrical configuration of the core, power distribution, source neutron characteristics, etc. For example, introducing a strong absorber around the source region can significantly reduce the source efficiency. On the other hand, inserting a fuel material between the target zone and the fuel blanket generally increases the efficiency. Thus, severe core disruptive accidents could result in a drastic change of \( \phi * \), and hence the BOP method cannot be used in such case. However, for a wide range of the coolant temperature and power distribution change, the variation of \( \phi * \) is rather small, within several percents. Furthermore, for a specific transient, the change of \( \phi * \) shows a very similar trend for different initial subcriticality levels. Therefore, the BOP method can be used for a relative comparison of the transient responses of ADS with different initial subcriticality levels, in spite of the possibly large errors in absolute values.

4.2 APPLICATION TO HYPER SYSTEM

In order to assess the impacts of the subcriticality level on the ADS safety, several important accidents were analyzed for the HYPER system with the BOP method. The three reactivity-related parameters, \( A, B, \) and \( C \) in the BOP method were estimated from the following relations[12]
\[
A = (\alpha_B + \alpha_E) \Delta T_{FC} \\
B = (\alpha_B + \alpha_E + \alpha_{LBE} + 2\alpha_R) \Delta T_c / 2 \\
C = (\alpha_B + \alpha_E + \alpha_{LBE} + \alpha_R)
\]
where \( \Delta T_{FC} \) is the difference between average fuel and coolant temperatures. Using the basic reactivity coefficients in Table A.II and \( \Delta T_{FC} = 175 ^\circ\text{C} \), the coefficients \( A, B, \) and \( C \) were estimated to be \(-97.3\,\text{pcm}\), \(-208.5\,\text{pcm}\), and \(-1.482\,\text{pcm}/^\circ\text{C}\), respectively.
Assuming that these coefficients are independent of the subcriticality level of the system, five types of transients were analyzed with the BOP method: insertion of reactivity (IOR), transient of current (TOC), chilled inlet temperature without scram (CIT-WS), loss of heat sink without scram (LOHS-WS), and loss of flow without scram (LOF-WS). All these analyses were performed assuming that the accelerator is not shut off. For each transient, the asymptotic power level and outlet coolant temperature were estimated at several subcriticality levels, and the results are summarized in Table I.

Table I. Effects of Subcriticality Level on Asymptotic Power and Coolant Outlet Temperature

<table>
<thead>
<tr>
<th>$k_{\text{eff}}$</th>
<th>IOR (270 pcm insertion)*</th>
<th>TOC (100% increase of current)</th>
<th>CIT-WS</th>
<th>LOHS-WS</th>
<th>LOF-WS (10% natural circulation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.995</td>
<td>$P = 1.41$; $T_{\text{out}} = 579 \degree\text{C}$</td>
<td>$P = 1.52$; $T_{\text{out}} = 598 \degree\text{C}$</td>
<td>$P = 1.20$; $T_{\text{out}} = 445 \degree\text{C}$</td>
<td>$T_{\text{out}} &gt;&gt; 1000 \degree\text{C}$</td>
<td>$P = 0.44$; $T_{\text{out}} = 1083 \degree\text{C}$</td>
</tr>
<tr>
<td>0.99</td>
<td>$P = 1.24$; $T_{\text{out}} = 551 \degree\text{C}$</td>
<td>$P = 1.66$; $T_{\text{out}} = 623 \degree\text{C}$</td>
<td>$P = 1.12$; $T_{\text{out}} = 431 \degree\text{C}$</td>
<td>$T_{\text{out}} &gt;&gt; 1000 \degree\text{C}$</td>
<td>$P = 0.54$; $T_{\text{out}} = 1254 \degree\text{C}$</td>
</tr>
<tr>
<td>0.98</td>
<td>$P = 1.13$; $T_{\text{out}} = 532 \degree\text{C}$</td>
<td>$P = 1.79$; $T_{\text{out}} = 644 \degree\text{C}$</td>
<td>$P = 1.07$; $T_{\text{out}} = 421 \degree\text{C}$</td>
<td>$T_{\text{out}} &gt;&gt; 1000 \degree\text{C}$</td>
<td>$P = 0.65$; $T_{\text{out}} = 1441 \degree\text{C}$</td>
</tr>
<tr>
<td>0.97</td>
<td>$P = 1.09$; $T_{\text{out}} = 525 \degree\text{C}$</td>
<td>$P = 1.85$; $T_{\text{out}} = 653 \degree\text{C}$</td>
<td>$P = 1.05$; $T_{\text{out}} = 418 \degree\text{C}$</td>
<td>$T_{\text{out}} &gt;&gt; 1000 \degree\text{C}$</td>
<td>$P = 0.71$; $T_{\text{out}} = 1551 \degree\text{C}$</td>
</tr>
<tr>
<td>0.96</td>
<td>$P = 1.06$; $T_{\text{out}} = 521 \degree\text{C}$</td>
<td>$P = 1.88$; $T_{\text{out}} = 659 \degree\text{C}$</td>
<td>$P = 1.05$; $T_{\text{out}} = 418 \degree\text{C}$</td>
<td>$T_{\text{out}} &gt;&gt; 1000 \degree\text{C}$</td>
<td>$P = 0.76$; $T_{\text{out}} = 1625 \degree\text{C}$</td>
</tr>
</tbody>
</table>

* corresponding to 1$\$ in case of IOR, it is assumed that the accelerator current and inlet coolant temperature do not change during the accident, and the increased power is absorbed in the secondary system. As expected, the asymptotic power and coolant outlet temperature decrease as the degree of subcriticality increases. Although the differences are rather small in the subcriticality range of $0.96 \leq k_{\text{eff}} \leq 0.98$, a larger subcriticality level is more favorable.

For the TOC event, the accelerator current was doubled to account for the large reserved current at BOC to compensate the large burnup reactivity swing in a TRU-fueled ADS core. The 100% increase of current roughly corresponds to $2\%\Delta k$ at an initial $k_{\text{eff}}$ of 0.98. For a relatively short or intermediate time period, the system response can be evaluated with the assumption that $\delta T_{\text{in}} = 0$. The results in Table I show that a higher $k_{\text{eff}}$ value is more favorable. This is due to the fact that the external source effects increase but the negative reactivity feedback effects decrease as the degree of subcriticality increases. Since the outlet temperature increase is rather significant in this accident, the inlet temperature would start to increase as time goes on. This inlet temperature increase in turn would decrease the power level. Thus, it is expected that the core would go slowly to a new equilibrium state with a lower power level and a higher inlet temperature in a long term. These results are contrary to the previous results of Gandini et al[3,4]; they concluded that a higher subcriticality level is desirable in case of TOC. These contradictory results are due to their incorrect approximation of Eq. (13); they
derived a simple approximate formula by neglecting the term $\delta i / i$ from Eq. (13),[3] but it cannot be neglected in general.

The asymptotic response for the CIT-WS accident was calculated with an inlet temperature decrease of 100 °C. The results in Table I show that the responses for the four subcriticality levels are all acceptable. It can be seen that a large subcriticality provides slightly better performance as in the case of IOR accident. This is because an inlet temperature decrease is basically equivalent to an insertion of a positive reactivity. However, the magnitude of the added reactivity is about half of the IOR case. It might be said that the CIT-WS accident does not cause a serious safety concern in ADS.

In the LOHS-WS case, the secondary heat exchanger is assumed to fail with a constant accelerator current. In this case, the outlet temperature would increase constantly since there is no heat loss from the primary system, despite the significant negative reactivity feedback due to the increased coolant temperature. Since the fission power would slowly decrease during the transient due to the negative reactivity feedback, the outlet temperate increase rate would decrease as time goes on. The temperature increase rate would be lower in a higher $k_{\text{eff}}$ core, since the negative reactivity feedback is more effective in the higher $k_{\text{eff}}$ regime. However, the outlet temperature would keep increasing unless the accelerator current is shut off.

The LOF-WS accident was analyzed with the assumption that the inlet temperature does not change while the coolant flow coasts down to a natural circulation. In this work, a 10% natural circulation was assumed for the LOF-WS event. The results in Table I show that a high $k_{\text{eff}}$ core provides a slightly better response than a low $k_{\text{eff}}$ core, since the negative reactivity feedback is more effective in the higher $k_{\text{eff}}$ regime. In this event, the coolant temperature rise is practically unacceptable in all the subcriticality levels. In a long term, the coolant temperature would further increase and the power level would gradually decrease.

These results show that the outlet temperature would be unacceptably high in both LOHS-WS and LOF-WS cases if the accelerator beam is not shut off. Therefore, the ADS should be equipped with a very reliable beam shutdown system. On the other hand, in case of a liquid target ADS with a beam window such as HYPER, there is some possibility that, during LOHS-WS and LOF-WS, the window might fail, leading to an beam shut-off before the coolant outlet temperature reaches an unacceptably high point. However, this kind of fail-safe effect might not be expected in a windowless target system, since the target system is almost independent of the reactor coolant system.

5. IMPACTS OF SUBCRITICALITY ON TRANSMUTATION PERFORMANCE

The essential objective of the transmutation of radioactive materials is to minimize the release of those radiotoxic nuclides into environment. In general, the transmutation of TRUs and LLFPs is based on a multiple recycling of the discharged material into the transmuter, since a complete transmutation is virtually impossible in a single fuel cycle. Therefore, the discharge burnup of TRUs and LLFPs should be maximized in order to minimize the loss of radiotoxicity to the environment during the recycling stage.

In an ADS transmuter with a fixed power level, the TRU transmutation rate, i.e, the amount of TRU consumed per cycle, is almost independent of its subcriticality level. However, since the fuel inventory is determined by the subcriticality level, the fuel discharge burnup rate that is inversely
proportional to the inventory depends on the degree of subcriticality. Therefore, if the subcriticality level is determined by adjusting the fuel loading only, a higher degree of subcriticality is desirable from the fuel burnup point of view since the fuel inventory decreases as the subcriticality level increases. In addition, for fixed amount of LLFP loading, the LLFP discharge burnup increases as the subcriticality level increases, since a higher subcriticality level provides more surplus neutrons due to the increased external source.

In order to quantify the impact of the subcriticality level on the transmutation performance, equilibrium cycle analyses were performed for the HYPER core using the REBUS-3 code system[6]. To investigate the LLFP transmutation performance, Tc-99 was transmuted by co-mingling metallic form of Tc-99 with fuel, since Tc-99 is one of the most problematic LLFPs due to its high mobility in a geological repository. (Note that Tc-99 is not loaded in the current design described in the Appendix.). It was assumed that Tc-99 is completely recovered during the reprocessing stage of the fuel and recycled into the core. For a systematic comparison of the Tc-99 transmutation performance, the inventory of Tc-99 was fixed at 124kg.

Table II compares the fuel and Tc-99 transmutation performances for three subcriticality levels in terms of transmutation rate and discharge burnup. The results show that a higher subcriticality provides better transmutation performances, although the differences are rather small. The fuel and Tc-99 discharge burnups are increased by ~8% and ~3%, respectively, when the \(k_{\text{eff}}\) value decreases from 0.99 to 0.97. The absolute transmutation rate of Tc-99 for the case of \(k_{\text{eff}}=0.99\) could be increased by loading more Tc-99. However, the increased Tc-99 inventory would necessarily result in lower discharge burnup rates of both the Tc-99 and fuel.

<table>
<thead>
<tr>
<th>Initial (k_{\text{eff}})</th>
<th>Initial inventory, kg Tc-99 / Fuel</th>
<th>Tc-99 transmutation</th>
<th>Fuel discharge burnup, a/o</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>%/cycle kg/cycle</td>
<td>Discharge Burnup, a/o</td>
</tr>
<tr>
<td>0.97</td>
<td>124 / 4644</td>
<td>1.99 2.47</td>
<td>14.9 21.84</td>
</tr>
<tr>
<td>0.98</td>
<td>124 / 4722</td>
<td>1.93 2.39</td>
<td>14.5 21.53</td>
</tr>
<tr>
<td>0.99</td>
<td>124 / 4801</td>
<td>1.87 2.32</td>
<td>14.0 21.23</td>
</tr>
</tbody>
</table>

6. SUMMARY AND CONCLUSIONS

It was attempted to estimate an optimal range of subcriticality of ADS systems from the operational and safety points of view. To devise a representative measure of the subcriticality level, the mathematical and physical implications of the effective multiplication factor and the source multiplication factor were reviewed. From this review, it was shown that the source multiplication factor has a limited applicability to indicate the subcriticality level from the safety point of view, while it is a good measure of subcriticality from the operational point of view. The effective multiplication factor was selected as a measure of subcriticality, since it indicates the degree of off-criticality of the system, independently of the external source distribution.

In order to determine the feasible domain of subcriticality, the safety requirements and the economical and technical constraints were examined. The minimum required subcriticality level was determined to preclude the potential for criticality as a result of operational or accidental reactivity insertions.
including the calculation and measurement uncertainties. The reactivity increase from the full power condition to the cold reloading state was used as the operational reactivity insertion. For the accidental reactivity insertion, the beam window failure and accidental control rod withdrawal (if control rods are used) were proposed to be considered from a practical point of view. On the other hand, the maximum allowable subcriticality level was determined by the system economy and the technical feasibility of the system. Specifically, the minimum multiplication factor required for an economic operation was employed as the economic constraint. As the technical constraint, the maximum beam power density that the beam window can hold out for a desired period was employed.

By applying these constraints on the HYPER system being developed at KAERI, the feasible domain of subcriticality was estimated to be $0.961 \leq k_{\text{eff}} \leq 0.988$ without including the uncertainties. From this bounding estimation, it was found that the current HYPER design does not satisfy the lower bound at EOC and hence design modifications need to be made. Within this feasible domain of subcriticality, a preliminary estimation of the optimal range of subcriticality was performed based on the safety and transmutation performances.

Using the balance of power method, asymptotic safety analyses were performed for five types of transients: insertion of reactivity (IOR), transient of current (TOC), chilled inlet temperature without scram (CIT-WS), loss of heat sink without scram (LOHS-WS), and loss of flow without scram (LOF-WS). The results showed that for LOHS-WS and LOF-WS accidents, the coolant outlet temperature would be unacceptably high regardless of the subcriticality level unless the accelerator beam is shut off. This result indicates that the ADS system should be equipped with a very reliable beam shutdown system. The results also showed that a higher $k_{\text{eff}}$ is favorable for the TOC accident, while a lower $k_{\text{eff}}$ is favorable for the IOR and CIT-WS accidents. However, the asymptotic response of the system to the IOR and CIT-WS accidents was relatively insensitive to the subcriticality level.

With respect to the transmutation performance, a higher subcriticality is favorable in terms of the fuel discharge burnup. The LLFP transmutation could also be done more efficiently in a higher subcriticality core. However, the transmutation performance is relatively insensitive to the subcriticality level. Therefore, this transmutation performance might not be a crucial parameter in determining the subcriticality level of an ADS.

These results suggest that it is desirable to maximize the effective multiplication factor within the feasible upper bound to preclude the potential for criticality. However, to determine this upper bound more realistically, detailed dynamic and safety analyses and uncertainty evaluations need to be performed.

**REFERENCES**

The HYPER (Hybrid Power Extraction Reactor) system is being studied at KAERI for the transmutation of both TRUs and long-lived fission products. HYPER is a 1,000 MWth LBE-cooled ADS with a single central spallation source. One GeV proton beam impinges on the LBE target and generates the spallation neutrons, and a beam window is used to separate the proton beam delivery tube (30 cm diameter) from an LBE target. A schematic configuration of the HYPER core is shown in Fig. A.1.

The core is loaded with a ductless fuel assembly containing TRU dispersion fuel pins, in which TRU-10Zr fuel particles are dispersed in Zr matrix. It is assumed that TRU elements are obtained by removing all fission products and 99.95% uranium from the PWR spent fuel of 33 GWD/MTU burnup. Consequently, uranium is about 4.6 w/o in the fuel composition. All the structural materials are HT-9 steel. The pitch-to-diameter (P/D) ratio of the fuel lattice is 1.5 and the active core height is 160 cm. In HYPER, a relatively high core height is adopted to maximize the multiplication efficiency of the external source.

In the ductless fuel assembly, 13 non-fuel rods are used as the tie rod to maintain the mechanical integrity of assembly. On the other hand, the hexagonal duct is used for the reflector and shield assemblies to adjust the coolant flow rate. The reflector assembly is composed of lead-filled HT-9 tubes to improve the neutron economy and to minimize the generation of radioactive materials. An auxiliary shutdown concept is adopted to provide a redundant shutdown mechanism which is
independent of the accelerator shutdown system. In case of emergency, a thick annulus containing B$_4$C absorber is inserted into the buffer zone to surround the spallation target zone. By absorbing the source neutrons directly, it can reduce the fission power to several percents even in the case the accelerator is not shut off.

To minimize the burnup reactivity swing, a half-year cycle length (140 full power days) is adopted, and B$_4$C burnable absorber is employed. For efficient depletion of the B-10 absorber, the burnable absorber is loaded only in the central part (92cm long) of the 13 tie rods of each assembly. However, the burnable absorber is not used in the innermost fuel ring since it can hamper the source multiplication efficiency. In the current design, the amount of the B$_4$C absorber was determined such that the burnup reactivity swing is about 3.0% $\Delta k$. An eight-batch fuel management scheme is adopted for the middle and outer core zones. The fuel residence time in the inner core zone is limited to seven cycles to limit the peak fast fluence. The power peaking is controlled by adjusting the TRU fraction in each fresh fuel assembly.

Table A.I summarizes the equilibrium-cycle performance parameters of the current HYPER design determined by the REBUS-3[6] equilibrium cycle analysis. In the REBUS-3 analysis, it is assumed that all the fuel elements are completely recovered and recycled into the core and 5% of the rare earth elements are carried over during the fuel reprocessing/fabrication processing. The reactivity change is over 5%$\Delta k$ in the HYPER core if the burnable absorber is not used. Table A.II provides some important reactivity coefficients and reactivity changes estimated using the DIF3D[15] code. More detailed analysis results can be found in Ref. 11.
Table A.I. Equilibrium Cycle Performance Parameters of HYPER Design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Inner Zone</th>
<th>Middle Zone</th>
<th>Outer Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Fuel Weight Fraction</td>
<td>39.1</td>
<td>45.3</td>
<td>48.7</td>
</tr>
<tr>
<td>Effective Multiplication Factor ($k_{eff}$)</td>
<td>BOC 0.980</td>
<td>EOC 0.951</td>
<td></td>
</tr>
<tr>
<td>Burnup Reactivity Loss (%Δk)</td>
<td>2.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core-Average Power Density (kW/l)</td>
<td>137</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-D Power Peaking Factor</td>
<td>BOC 1.67</td>
<td>EOC 1.95</td>
<td></td>
</tr>
<tr>
<td>Average Fuel Discharge Burnup (a/o)</td>
<td>21.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average B-10 Discharge Burnup (a/o)</td>
<td>46.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net TRU Consumption Rate (kg/year)</td>
<td>290</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equilibrium Loading (kg/year)</td>
<td>LWR TRU 290</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Recycled TRU 1036</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total TRU 1326</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heavy Metal Inventory (kg)</td>
<td>BOC 4642</td>
<td>EOC 4497</td>
<td></td>
</tr>
</tbody>
</table>

Table A.II. Reactivity Coefficients and Changes of HYPER Design

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBE coolant density variation, $\alpha_{LBE}$</td>
<td>+0.045 pcm/°C</td>
</tr>
<tr>
<td>Fuel Doppler effect at nominal temperature, $\alpha_{D}$</td>
<td>−0.031 pcm/°C</td>
</tr>
<tr>
<td>Radial core expansion, $\alpha_{R}$</td>
<td>−0.971 pcm/°C</td>
</tr>
<tr>
<td>Axial fuel element expansion, $\alpha_{E}$</td>
<td>−0.525 pcm/°C</td>
</tr>
<tr>
<td>Reactivity change due to the window failure</td>
<td>+753 pcm</td>
</tr>
<tr>
<td>LBE void reactivity (in active core only)</td>
<td>+2,745 pcm</td>
</tr>
<tr>
<td>Reactivity change due to complete coolant loss</td>
<td>−24,834 pcm</td>
</tr>
</tbody>
</table>

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