

Stodolsky's Theorem and Neutrino Oscillation Phases—for pedestrians

By

Harry J. Lipkin

Department of Particle Physics Weizmann Institute of Science, Rehovot 76100, Israel

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact
Sciences, Tel Aviv University, Tel Aviv, Israel

Argonne National Laboratory, High Energy Physics Division, Argonne, IL 60439 USA

December 9, 2002

**Work supported in part by the U.S. Department of Energy, Division of High Energy Physics, under
Contract W-31-109-ENG-38 and by a grant from US-Israel Bi-National Science Foundation.*

Argonne National Laboratory, with facilities in the states of Illinois and Idaho, is owned by the United States Government and operated by The University of Chicago under the provisions of a contract with the Department of Energy.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor The University of Chicago, nor any of their employees or officers, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of document authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof, Argonne National Laboratory, or The University of Chicago.

Available electronically at <http://www.doe.gov/bridge>

Available for a processing fee to U.S. Department of Energy and its contractors, in paper, from:

U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831-0062
phone: (865) 576-8401
fax: (865) 576-5728
email: reports@adonis.osti.gov

Stodolsky's Theorem and Neutrino Oscillation Phases – for pedestrians

Harry J. Lipkin *

Department of Particle Physics Weizmann Institute of Science, Rehovot 76100, Israel

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Tel Aviv, Israel

High Energy Physics Division, Argonne National Laboratory, Argonne, IL 60439-4815, USA

lipkin@hep.anl.gov

Abstract

Neutrino oscillations are experimentally observable only as a result of interference between neutrino states with different masses and THE SAME ENERGY. All interference effects between neutrino states having different energies are destroyed by the interaction between the incident neutrino and the neutrino detector. Erroneous results are frequently obtained by neglecting the neutrino-detector interactions.

Stodolsky [1] has given a very simple answer to the confusion that still arises in discussions of the phase of neutrino oscillations. The relevant literature producing this confusion has recently been summarized and clarified [2]. The purpose of this note is to support this excellent analysis [2] without engaging in a direct debate against the confusing articles and also to present a “pedestrian” version of Stodolsky’s work which is hopefully understandable to students and experimentalists.

The detection of a neutrino always involves its interaction with a detector that is part of an environment described by a density matrix in which the energy is diagonal. Unless this interaction with the environment is turned off, and no experiment can do this, all relative

*Supported in part by grant from US-Israel Bi-National Science Foundation and by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38.

phase information between neutrino states with different energies is destroyed.

The question is not whether states of the same momentum and different energies are coherent, states of the same energy and different momentum or states of the same velocity. There have been many irrelevant arguments about these issues. But states with different ENERGIES ARE NEVER COHERENT in any realistic experiment. States of the same energy and different momenta can be coherent, but may not be. This depends upon the way the measurement is made. But states with different energies can not be coherent.

This discussion refers only to neutrino detectors, The usual detector is a nucleon, which changes its state after absorbing a neutrino and emitting a charged lepton, and is initially either in an energy eigenstate or in a statistical mixture in thermal equilibrium with its surroundings. No neutrino detector has ever been prepared in a coherent mixture of energy eigenstates and no such detector has been proposed for future experiments .

All arguments about Lorentz invariance are irrelevant. The detector chooses a particular Lorentz frame where the detector is at rest and described by a density matrix in which the energy is diagonal and no interference between states of different energies can be observed.

Most treatments do not consider at all the quantum mechanics of the detector. Since the detector is a quantum system (e.g. a nucleon) which undergoes a transition together with the neutrino-to-charged-lepton transition, and the initial and final states of the detector are not measured, the transition probability is the square of the transition matrix element for the whole system, averaged over detector initial states and summed over detector final states. This immediately kills all interference between neutrino states with different energies as they are accompanied by different detector states which must have different energies because energy is conserved in the process. The detector states with different energies are orthogonal to one another and all interference terms between them vanish because of this detector orthogonality.

In this context the “factor-of-two” arguments are seen to be missing an essential point in the actual neutrino oscillation experiments; namely the role of the detector as a quantum-mechanical system entangled with the neutrino.

The standard textbook neutrino-oscillation wave function, a coherent linear combination of states with different energies, is not found in real experiments. Thus considerable confusion remains even though coherence, interference and dephasing have been extensively discussed and clarified [1,3,4,5,6,7,8,9,10,11,12]. Elementary quantum mechanics and quantum statistical mechanics tell us that the components of the density matrix describing a neutrino detector and having different energies are never coherent [1], while neutrino components with different masses and different momenta must be coherent to cancel components with the wrong flavor just outside the neutrino source.. This coherence between source states having the same energy and different momenta can produce coherence between neutrino states with the same energy and different masses.

This physics is illustrated in detail in a toy model [13] for the detection of a neutrino as a transition between an initial state of a neutrino and a detector and a final state of a muon and the same detector. The wave function for the initial state of neutrino and detector is

$$\Psi_i(\nu, D) = \sum_{k=1}^{N_\nu} \left| \nu(E_\nu, m_k, \vec{P}_k), D_i(E_i) \right\rangle \quad (0.1)$$

where N_ν is the number of neutrino mass states, E_ν , m_k and \vec{P}_k denote the neutrino energy, mass and momentum and $D_i(E_i)$ is the initial state of the detector with energy E_i . If the detector is a muon detector the final detector state after neutrino absorption is

$$\Psi_f(\mu^\pm, D) = \sum_{k=1}^{N_\nu} \left| \mu^\pm(E_\mu, \vec{P}_\mu), D_{kf}^\mp(E - E_\mu) \right\rangle \quad (0.2)$$

where E_μ and \vec{P}_μ denote the muon energy and momentum, D_{kf}^\mp is the final detector state produced in the ‘‘path k ’’; i.e. after the absorption of a neutrino with mass m_k and emission of a μ^\pm , and $E = E_\nu + E_i$ is the total energy which is conserved in the transition.

The transition in the detector occurs on a nucleon, whose co-ordinate is denoted by by \vec{X} , and involves a charge exchange denoted by the isospin operator I_\mp and a momentum transfer $\vec{P}_k - \vec{P}_\mu$. The detector transition matrix element is therefore given by

$$\langle D_{kf}^\mp | T^\mp | D_i \rangle = \langle D_{kf}^\mp | I_\mp e^{i(\vec{P}_k - \vec{P}_\mu) \cdot \vec{X}} | D_i \rangle \quad (0.3)$$

The overlap between the final detector wave functions after the transitions absorbing neutrinos with masses m_k and m_j is then

$$\langle D_{kf}^{\mp} | D_{jf}^{\mp} \rangle = \langle D_i | e^{i(\vec{P}_j - \vec{P}_k) \cdot \vec{X}} | D_i \rangle \quad (0.4)$$

If the quantum fluctuations in the position of the active nucleon in the initial state of the detector are small in comparison with the oscillation wave length, $\hbar/(\vec{P}_j - \vec{P}_k)$,

$$|\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i | |\vec{X}^2| | D_i \rangle \ll 1 \quad (0.5)$$

$$\langle D_{kf}^{\mp} | D_{jf}^{\mp} \rangle \approx 1 - (1/2) \cdot |\vec{P}_j - \vec{P}_k|^2 \cdot \langle D_i | |\vec{X}^2| | D_i \rangle \approx 1 \quad (0.6)$$

There is thus effectively a full overlap between the final detector states after absorption of different mass neutrinos, and a full coherence between the neutrino states with the same energy and different momenta.

The total energies of the final muon and detector produced after absorption of neutrinos with different energies are different. These muon-detector states are thus orthogonal to one another and there is no coherence between detector states produced by the absorption of neutrinos with different energies.

It is a pleasure to thank Maury Goodman, Yuval Grossman, Boris Kayser, Lev Okun, and Leo Stodolsky for helpful discussions and comments.

REFERENCES

- [1] Leo Stodolsky, Phys. Rev. D58 (1998) 036006
- [2] L. B. Okun et al. hep-ph/0211241
- [3] H.J. Lipkin, Neutrino Oscillations for Chemists, Lecture Notes (unpublished).
- [4] H.J. Lipkin, Lecture Notes on Neutrino Oscillations for a Quantum Mechanics course (unpublished).
- [5] H.J. Lipkin, Neutrino Oscillations and MSW for Pedestrians, Lecture Notes (unpublished).
- [6] B. Kayser, Phys. Rev. D 24 (1981) 110.
- [7] T. Goldman, hep-ph/9604357.
- [8] H.J. Lipkin, Phys. Lett. B 348 (1995) 604.
- [9] M.M. Nieto, hep-ph/9509370, Hyperfine Interactions 100 (1996) 193
- [10] Yuval Grossman and Harry J. Lipkin, Phys. Rev. D55 (1997) 2760
- [11] A.D. Dolgov, A.Yu. Morozov, L.B. Okun, and M.G. Schepkink, *Nucl. Phys.* **502** (1997) 3
- [12] Harry J. Lipkin, hep-ph/9901399, Weizmann Preprint WIS-98/31/Dec-DPP, Tel Aviv University preprint TAUP 2537-98, Argonne Preprint ANL-HEP-CP-98-126, available in Proceedings of the Europhysics Neutrino Oscillation Workshop (NOW'98) 7-9 September 1998. Amsterdam. Published in <http://www.nikhef.nl/pub/conferences/now98/>
- [13] H.J. Lipkin, hep-ph/9907551, Phys. Lett. B 477 (2000) 195