Initial development of a high temperature life prediction method directly accounting for variability in material properties

Applied Materials Division

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ABSTRACT

This report presents the initial development of a life prediction method that accounts for the variability of the material properties in Grade 91 steel. We account for material variability by both fitting a variable 3-parameter Weibull distribution to the rupture data of the material and by quantifying the effect that the creep deformation properties have on stresses via a Monte Carlo approach. To ensure a reasonable timeframe when applying Monte Carlo to the creep stress, we propose a methodology that models the material as an extremely viscous Stokes flow with a non-Newtonian viscosity, therefore solving the problem in a single step. We use the final probabilistic model for structural failure developed in this document to evaluate a Flat Head Vessel designed with the Section III, Division 5 ASME Boiler Pressure and Vessel Code (BPVC) rules with a 100,000h design life and assess its probability of premature failure.

ACRONYMS AND ABBREVIATIONS

Acronym	Definition
AD	Automatic Differentiation
ASME	American Society of Mechanical Engineers
BC	Boundary Condition
BPVC	Boiler Pressure and Vessel Code
CDF	Cumulative Distribution Function
CI	Confidence Interval
FHV	Flat Head Vessel
GR91	Grade 91 Steel (9Cr-1Mo-V)
GP	Gaussian Process
INS	Incompressible Navier Stokes
LMP	Larson-Miller Parameter
LR	Linear Regression
MC	Monte Carlo
MCR	Minimum Creep Rate
MOOSE	Multiphysics Object-Oriented Simulation Environment
PDF	Probability Distribution Function
PI	Prediction Interval
POF	Probability of Failure
PSPG	Pressure-Stabilized Petrov-Garlekin
RBF	Radial Basis Function
SUPG	Streamline-Upwind Petrov-Galerkin

TABLE OF CONTENTS

Abstract	i
Acronyms and abbreviations	iii
Table of Contents	v
List of Figures	vii
List of Tables	ix
1 Introduction	1
2 Methodology	3
2.1 Experimental Data Fitting	3
2.1.1 Creep Life prediction	3
2.1.2 LMP Distribution Fit	4
2.1.3 Power Law Creep Parameters	6
2.2 Modeling of Creep	8
2.2.1 Geometry and boundary conditions	8
2.2.2 Stokes Modeling	8
2.2.2.1 MOOSE INS module	8
2.2.2.2 Non-Newtonian viscosity	10
2.2.2.3 Stress calculations	10
2.2.3 Modeling and Validation	12
3 Results	15
3.1 Stress Variability through Monte Carlo	15
3.2 Probability of rupture time	16
3.3 Assessing the robustness of the ASME primary load design process	17
4 Conclusions and Future Work	19
Acknowledgements	21
References	23

LIST OF FIGURES

Figure 1. Overview of processes needed to generate a predictive statistical method.....	3
Figure 2. Gaussian Process fit of GR91 experimental data.	4
Figure 3. PDF (left) and CDF (right) of the LMPs thorough the entire stress range of GR91. The experimental data is plotted (blue scatter points) alongside the GP fit (red lines).	6
Figure 4. Linear Regression fit of the MCR at 600°C.....	7
Figure 5. GR91 Flat Head Vessel (FHV) geometry and Boundary Conditions (BCs).....	8
Figure 6. Comparison of radial stresses obtained from the Stokes model (left) and the Structural model (center). The stress profile quantitatively matches, as seen in the graph (right).	12
Figure 7. Von Mises stress distribution in the FHV, for a power-law creep evaluation @ 600°C	15
Figure 8. Histogram (left) and CDF (right) of the maximum Von Mises stresses of all Monte Carlo iterations. The fitted Gaussian distribution as well as its CDF is plotted in orange.....	16
Figure 9. PDF (left) and CDF (right) of the time to rupture for GR91 at 600°C. The red line shows the design life of the FHV.	17

LIST OF TABLES

Table 3.1. Premature failure probabilities for designs to incorrect allowable stresses.....	18
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1 Introduction

There is a growing interest in the nuclear community in including the variability of material properties in the design and evaluation of component life, so that the engineer can have an actual statistical model for premature structural failure informed by full distributions instead of a single pass-fail model informed by lower/upper bounds.

One of the challenges in developing a statistical methodology for high temperature components with time-dependent material properties is the need for transient analysis. For example, Monte Carlo methods require a large amount of component evaluations with variable material properties. However, given the relatively long time needed to perform a single evaluation, conducting this set of evaluations becomes prohibitively expensive.

Another challenge is actually the uncertainty of creep rupture data with a representative statistical model. Traditionally, rupture stress/time/temperature correlations used on the Larson-Miller parameter or similar empirical creep parameters to obtain lower and upper prediction bounds, assuming a normal distribution of the data. However, not all materials follow a simple trend in the Larson-Miller parameter, and oftentimes the rupture data does not follow a normal distribution.

To eliminate the numerical cost of transient simulation, we present in this work a one-step methodology that quickly calculates the stabilized, steady-state creep stresses in the component with a steady-state analysis. We model the material as a steady-state Stokes flow with a non-Newtonian viscosity, instead of the usual practice of solving the elastic-creep model of the component through several time steps. Modeling steady creep as flow allows us to obtain the stresses in about a minute per component simulation, which is 30 times faster than the traditional approach. Therefore, the total time required to evaluate the variability of the material properties through the Monte Carlo method scales down to a manageable timeframe.

To properly quantify the uncertainty of the rupture data, we fit a variable 3-parameter Weibull distribution to the rupture data, in which each of the 3 Weibull parameters are a function of rupture stress. This stress-dependence allows Weibull to follow the overall trend of the rupture data regardless of its shape and distribution, therefore making it more flexible than the usual regression fit with normal distributions. Finally, we validate the results of this uncertainty quantification with a Gaussian Process fit.

To evaluate and compare the proposed life prediction method against the traditional ASME life prediction approach, we evaluate a Flat Head Vessel designed with the ASME Section III, Division 5 rules to a 100,000h design life. The results show that the design has a very low probability of premature failure (i.e. failure before the design life), which translates into a potentially wide design margin. Therefore, this method shows the potential of improving the design of components based on a quantifiable design margin that can be used by the engineer to make more informed decisions.

2 Methodology

We follow the overall procedure described in Figure 1 to generate a life prediction method that can ultimately be used to evaluate the probability of failure in the component. The first step in the procedure requires the probabilistic description of creep properties in Grade 91 steel (GR91, also known as 9Cr-1Mo-V) which then informs the Stokes analysis in the second step. The second step calculates steady-state creep stresses by modeling the material as a non-Newtonian viscous fluid. The third step requires the probabilistic evaluation of creep-rupture test data as a function of stress, which we approach in this methodology through 3-parameter Weibull fit and we validate with Gaussian fitting. The results from the three previous steps yield the final statistical model for life prediction with uncertainty, which is addressed in the results section.

Figure 1. Overview of processes needed to generate a predictive statistical method.

2.1 Experimental Data Fitting

2.1.1 Creep Life prediction

We used experimental data of GR91 to investigate the relationships between stress, temperature, and rupture life in the material. To bring the 3-variable problem down to a 2-variable problem, the temperature T and rupture life t were jointly represented with the Larson Miller Parameter (LMP) as:

$$LMP = \frac{T}{1000} (\log(t) + 33.1) \quad (1)$$

With the problem now being reduced to two variables, we used Gaussian Process (GP) modeling [1] to fit an optimized relationship between stress and LMP. There are several

major advantages to using GP modeling: (1) the user does not need to define an explicit trend equation during fitting, (2) the model obtains the most optimal fit, and (3) it accounts for uncertainty propagation for predictions beyond the experimental data.

In this work, we used GPy, a Gaussian process framework available in Python as a module [2]. To fit the LM model to the test data, we used a 1-dimensional radial basis function (RBF) kernel with an initial length scale of 10 and an initial variance of 4. The GP fit also needs a likelihood and an inference method, both of which are set as Gaussian. We set the variance of the likelihood to be 0.05 and the inference method to be exact.

The GPy module fitted a response based on training data, which was comprised of any experimental data with a rupture life below 10,000 hours. The GPy module automatically optimized the RBF kernel's shape, which was calculated to have an optimal variance of 2.29 and an optimal length scale of 9.21. The optimized kernel was then used to measure the similarity between data points to ultimately obtain a prediction of the value for an unseen point. The prediction contained both the estimate of the value (the fitted response) as well as uncertainty information (the 95% prediction intervals). Figure 2 shows the final GP fit of the GR91 training data alongside the scatter plot of the entire dataset, which also includes experimental data with rupture lives beyond 10,000 hours. The GP fit thus proves to be an adequate representation of the relationship between stress and LMP in GR91.

Figure 2. Gaussian Process fit of GR91 experimental data.

2.1.2 LMP Distribution Fit

Given the optimal solution given by the GP fit, we are interested next in describing the uncertainty of LMP as a function of stress. To do so, we fit a 3-parameter Weibull distribution on the experimental LMP distribution at each stress state. We also investigated other types of distribution models (2-parameter Weibull, Gaussian, etc), and compared their goodness of fit using the Kolmogorov-Smirnov test. In this test, a high p-value indicates a good distribution fit. Any p-value below 0.05 means a rejection of the chosen distribution fit. Out of all

the distributions, the 3-parameter Weibull fit consistently showed a p-value above 0.95 (the maximum value attainable is 1.0). All other distribution fits exhibited a p-value between 0.2 and 0.4, indicating that the 3-parameter Weibull fit is the best representation of the LMP distributions.

Furthermore, to ensure that enough data points are included at each Weibull distribution fit, the stresses were evaluated in clusters of 10 MPa. For each stress, we fitted the following probability density function (PDF):

$$f(x) = \frac{\kappa}{\lambda} \left(\frac{x - \theta}{\lambda} \right)^{\kappa-1} \exp \left[- \left(\frac{x - \theta}{\lambda} \right)^{\kappa} \right] \quad (2)$$

If integrated, its cumulative density function (CDF) becomes:

$$F(x) = 1 - \exp \left[- \left(\frac{x - \theta}{\lambda} \right)^{\kappa} \right] \quad (3)$$

In both equations, κ is the shape parameter, λ is the scale parameter, and θ is the location parameter of the Weibull distribution. We used the statistics module in Python's Scipy [3] to obtain the parameters that best fit the experimental data.

Once we obtained all parameters at each rupture stress, we assessed different types of correlations between stress and Weibull parameters. We found that the best way to describe their correlation was through a linear relationship, where a cutoff between low-stress behavior and high-stress behavior can be made at 100 MPa for the shape κ and the scale λ parameters:

$$\begin{aligned} \kappa &= \begin{cases} -0.0108 \sigma + 5.4453, & \sigma < 100 \text{ MPa} \\ -0.0046 \sigma + 4.1431, & \sigma \geq 100 \text{ MPa} \end{cases} \\ \lambda &= \begin{cases} -0.0416 \sigma + 4.9567, & \sigma < 100 \text{ MPa} \\ 0.0002 \sigma + 0.9221, & \sigma \geq 100 \text{ MPa} \end{cases} \\ \theta &= -0.0224 \sigma + 34.121 \end{aligned}$$

Using these linear relationships, we then calculated the PDFs and the CDF through the entire stress range of the material using Equation 2 and Equation 3. We see that the distributions show a very good agreement with both the experimental data and the optimized Gaussian fit, as seen in Figure 3. It should be noted that the CDF is a representation of the

probability of failure (POF) in the material, where 0 ensures no chance of failing, and 1 guarantees failure.

Figure 3. PDF (left) and CDF (right) of the LMPs thorough the entire stress range of GR91. The experimental data is plotted (blue scatter points) alongside the GP fit (red lines).

Ultimately, this mathematical description of the LMP Weibull distribution as a function of stress will be used to describe the marginal probability P of the time to rupture given a specified rupture stress $P(t|\sigma_i)$.

2.1.3 Power Law Creep Parameters

To obtain the power-law creep parameters that describe the creep deformation behavior of GR91, we fitted a linear regression to experimental Minimum Creep Rate (MCR) data of the

material at 600°C, as seen in Figure 4. The slope and the intercept of the fitted linear regression are analogous to the power-law creep parameters of the material, where:

$$\log(\text{MCR}) = a \log(\sigma) + b$$

and the power-law creep parameters n , A are:

$$n = a$$

$$A = 10^b$$

We also calculated the standard deviation of each parameter as well as the 95% confidence intervals (CI). Overall, the mean and CIs of the parameters become:

$$a \text{ (or } n\text{): } \mu = 8.956, \quad \text{CI} = (5.821, 12.091)$$

$$b: \quad \mu = -25.405, \quad \text{CI} = (-32.202, -18.608)$$

$$A: \quad \mu = 3.935 \times 10^{-26}, \quad \text{CI} = (6.277 \times 10^{-33}, 2.466 \times 10^{-19})$$

These power-law creep parameters were fitted with a Gaussian distribution described by their mean and standard deviation. These distribution fits are used in Section 3.1 for a Monte Carlo analysis of the material parameters. Any combination of these power-law creep parameters within 2 standard deviations of their mean will yield a linear regression fit bounded by the prediction intervals in Figure 4.

Figure 4. Linear Regression fit of the MCR at 600°C.

2.2 Modeling of Creep

2.2.1 Geometry and boundary conditions

Using the primary load design rules in the ASME Boiler Pressure and Vessel Code (BPVC), Section III, Division 5 [4], we designed a flathead vessel (FHV) geometry for a design life of 100,000h at 600°C and an inner pressure of 1 MPa. The axisymmetric RZ geometry, as well as the overall boundary conditions (BCs), are shown in Figure 5.

Figure 5. GR91 Flat Head Vessel (FHV) geometry and Boundary Conditions (BCs).

2.2.2 Stokes Modeling

Our approach to obtain the stabilized creep behavior in GR91 is by modeling the material as a very slow fluid with a very thick and variable viscosity. To model this type of fluid with strong viscous forces, we resort to modeling the fluid as a steady Stokes flow with a non-Newtonian viscosity. The power-law creep parameters are used to determine the viscosity of the fluid, and the Stokes flow equations are used to solve for the final stress distributions in the FHV.

2.2.2.1 MOOSE INS module

To solve the analogous creep problem using fluid mechanics, we use the Navier-Stokes module [5] inside MOOSE [6]. To calculate the Jacobians automatically (while sacrificing some computing speed), we use the Automatic Differentiation (AD) version of the module. This module has two variants of the governing Incompressible Navier-Stokes (INS) equations, the so-called “traction” and “Laplace” forms. The main difference between these forms is that the Laplace form assumes a constant viscosity. Given that we are interested in

modeling the creep behavior by modeling a non-Newtonian fluid with variable viscosity, we apply to the Traction form of the INS equations:

$$R(\vec{U}) \equiv \left[\begin{array}{c} \rho \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \text{grad}) \vec{u} \right) - \text{div } \sigma - \vec{f} \\ \text{div } \vec{u} \end{array} \right] = 0 \quad (4)$$

Where $\vec{U} \equiv (\vec{u}, p)$ is a vector composed of the velocity vector $\vec{u}$ and the pressure p , both of which are unknowns in our problem. Additionally, ρ is the (constant) fluid density, $\vec{f}$ is a body force per unit volume, and σ is the total stress tensor.

If we assume that the inertial forces are negligible, that is, that the transient term “ $\partial \vec{u} / \partial t$ ” and the convective term “ $(\vec{u} \cdot \text{grad}) \vec{u}$ ” can be ignored, we end up with the Stokes equations:

$$R(\vec{U}) \equiv \left[\begin{array}{c} \text{div } \sigma + \vec{f} \\ \text{div } \vec{u} \end{array} \right] = 0 \quad (5)$$

Each term in Equation 4 can be activated or deactivated by using Boolean flags in MOOSE’s input file. To solve the Stokes problem in Equation 5 using the AD version of the module, we simply do not call the kernels that calculate the momentum terms.

We need to also take into consideration two special instruments that the Navier-Stokes module uses when solving the flow problem. Firstly, the INS module solves this traction form of the Stokes equations, plus initial and boundary conditions, using a stabilized Petrov–Galerkin Finite Element Method (FEM). To solve issues with an indeterminate pressure p , the Navier-Stokes module applies the divergence theorem to integrate by parts any terms in the weak formulation that involve pressure (usually the viscous term and the pressure BC term). If the pressure is not integrated by parts, then the pressure needs to be “pinned”, that is, the user needs to specify at least one value of the pressure at some point of the domain. Therefore, we need to determine whether or not the problem will be integrated by parts by using the Boolean flag “integrate_p_by_parts” in the input file, and “pin” the pressure if it is not integrated by parts.

Secondly, we need to address the saddle point problems that arise during the discretization of Stokes flow. To circumvent these instabilities, the Navier-Stokes module uses the Pressure-Stabilized Petrov-Garlekin (PSPG) method. In the case of Stokes flow, the PSPG stabilization parameter is given by:

$$\tau_{PSPG} = \beta \frac{h^2}{4\nu} \quad (6)$$

Where h is the element size, $\nu = \mu / \rho$ is the kinematic viscosity, and β is a tunable parameter, which optimally is $\sim 1/3$. To generalize the PSPG method for problems involving advec-

tion, the Navier-Stokes module uses the expression of the Streamline-Upwind Petrov-Galerkin (SUPG) method instead, which is originally intended to stabilize advection-dominated flows:

$$\tau_{SUPG} = \alpha \left[\left(\frac{2}{\Delta t} \right)^2 + \left(\frac{2|\vec{u}|}{h} \right)^2 + 9 \left(\frac{4\nu}{h^2} \right)^2 \right]^{-1/2} \quad (7)$$

Where Δt is the discrete timestep size, $\nu = \mu/\rho$ is the kinematic viscosity, and α is a tunable parameter that ranges from 0 to 1. Equation 7 is basically equivalent to Equation 6 if a steady-state Stokes flow is assumed:

$$\tau_{SUPG} = \alpha \left[3 \left(\frac{4\nu}{h^2} \right) \right]^{-1} = \alpha \frac{1}{3} \frac{h^2}{4\nu} \quad (8)$$

2.2.2.2 Non-Newtonian viscosity

To model power law creep in the material, we need to model a flow with non-Newtonian viscosity. The non-Newtonian viscosity is described as:

$$\mu = \mu_0 \dot{\gamma}_{eff}^{m-1} \quad (9)$$

Where μ_0 is the initial viscosity, m is the thickening/thinning exponent (note that at 1 the viscosity becomes Newtonian), and where $\dot{\gamma}_{eff}$ is the effective shear rate, which in radial RZ coordinates is defined as:

$$\dot{\gamma}_{eff} = \sqrt{\frac{2}{3} \left(2 \left[\left(\frac{\partial v_r}{\partial r} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 + \left(\frac{v_r}{r} \right)^2 \right] + \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right]^2 \right)} \quad (10)$$

Since MOOSE does not support the description of non-Newtonian viscosity, we developed an AD material object called "*INSADNonNewtonianMu*" that calculates the non-Newtonian viscosity of the material using Equation 9 and Equation 10.

2.2.2.3 Stress calculations

Generally, the stress tensor is defined as:

$$\sigma = -PI + \tau \quad (11)$$

Where P is the pressure, I is the identity matrix and τ is the viscous stress tensor (shear stresses) in the material, which is defined as:

$$\tau = \mu \dot{\gamma} \quad (12)$$

The shear rate tensor $\dot{\gamma}$ (in radial RZ coordinates) is defined as:

$$\dot{\gamma} = \begin{bmatrix} 2 \frac{\partial v_r}{\partial r} & 0 & \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \\ 0 & 2 \left(\frac{v_r}{r} \right) & 0 \\ \frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} & 0 & 2 \frac{\partial v_z}{\partial z} \end{bmatrix} \quad (13)$$

To obtain an analogous version of the power-law creep model, we need to substitute the viscosity μ in Equation 12 with the definition of non-Newtonian viscosity in Equation 9, therefore approximating τ to:

$$\tau_{eff} \cong \mu_0 \dot{\gamma}_{eff}^m \quad (14)$$

Solving for the shear rate, we obtain the following relationship:

$$\dot{\gamma}_{eff} = \left(\frac{\tau_{eff}}{\mu_0} \right)^{1/m} \quad (15)$$

Which is analogous to the usual power-law definition:

$$\dot{\gamma}_{eff} = A \tau_{eff}^n \quad (16)$$

Where:

$$\begin{aligned} n &= 1/m \\ A &= \mu_0^{-n} \end{aligned}$$

Therefore, the main input of the Stokes model are the power-law creep parameters A and n , which in turn define the parameters μ_0 and n that describe the non-Newtonian viscosity

of the material. The power-law creep parameters are therefore the main inputs of the “*IN-SADNonNewtonianMu*” material object in MOOSE.

2.2.3 Modeling and Validation

We generated both the geometry and the mesh of the FHV with Trelis, which is a meshing pre-processor based on Sandia National Laboratory’s CUBIT mesher. The transition between the flathead and the wall as well as the wall itself underwent heavy mesh refinement. The final mesh contained 43,680 QUAD4 elements and 45,167 nodes.

We modeled the BCs seen in Figure 5 using the incompressible Navier Stokes module described in section 2.2.2, with a non-Newtonian viscosity where $A = 1.81 \times 10^{-26}$ and $n = 9.17$ for GR91 at 600°C. Since the BC pre-determines pressure values at some nodes, therefore “pinning” the pressure, there was no need to integrate the pressure by parts. However, the user will activate the integration by parts if no pressure is applied as a BC. To improve stabilization of the model, we set the α value in Equation 7 to 0.1.

To validate the results obtained from the steady-state Stokes model, we also modeled the FHV mesh and BCs using DEER, which is a set of various mechanics modules for the MOOSE framework (<https://github.com/Argonne-National-Laboratory/deer>). For the structural model of the FHV, we modeled the creep behavior of GR91 at 600°C using a power-law model where $A = 1.81 \times 10^{-26}$ and $n = 9.17$. The transient model was run for $\sim 300,000$ h (approximately 30 years), and the resulting stress distributions were compared to those obtained from the Stokes model, as seen in Figure 6.

Figure 6. Comparison of radial stresses obtained from the Stokes model (left) and the Structural model (center). The stress profile quantitatively matches, as seen in the graph (right).

The stress profile of the steady-state Stokes model matches the stress profile of the creep structural model once the stresses stabilized. Therefore, the Stokes model is validated to properly represent the steady-state creep behavior of materials, with the main benefit being its speed: while the structural model takes ~ 30 minutes to converge to the steady result, the stokes model converges quadratically in ~ 60 seconds. Therefore, the Stokes

model is an ideal tool for solving a large number of problems in a reasonable amount of time. We therefore use the steady-state Stokes model to model the effect of material parameter variability on the stress response of the FHV via the Monte Carlo (MC) method.

3 Results

3.1 Stress Variability through Monte Carlo

To evaluate the stress variability in the FHV through Monte Carlo method, we randomly selected 500 different combinations of the power-law creep parameters from the distribution described in Section 2.1.3, and use them to instantiate 500 unique steady-state Stokes models. For each result, we calculate the Von Mises stresses as:

$$\sigma_{VM} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \quad (17)$$

where σ_1 , σ_2 , σ_3 are the three principal stresses in the material. The overall distribution of the Von Mises stresses σ_{VM} in the FHV can be seen on Figure 7. While the quantitative value of the stresses differs for each iteration, the general distribution of the stresses does not change. That is, the high stresses are always concentrated at the thin wall of the FHV throughout all the Monte Carlo iterations.

Figure 7. Von Mises stress distribution in the FHV, for a power-law creep evaluation @ 600°C

We extracted the maximum Von Mises stress from each Monte Carlo iteration, and obtained their overall distribution, as seen in Figure 8. The histogram of the stresses matched a

Gaussian distribution with a mean value of 46.24 MPa and a standard deviation of 0.11 MPa, as verified by their CDFs.

Figure 8. Histogram (left) and CDF (right) of the maximum Von Mises stresses of all Monte Carlo iterations. The fitted Gaussian distribution as well as its CDF is plotted in orange.

The calibrated Gaussian distribution therefore describes the marginal probability of stress $P(\sigma_i)$ in the FHV given a variability of the material properties of GR91 at 600°C.

3.2 Probability of rupture time

To investigate the marginal probability of time to rupture, we need to evaluate the joint probability of time and stress throughout the entire stress range:

$$P(t) = \sum_i P(t \cap \sigma_i) \quad (18)$$

The multiplication rule allows us to rewrite Equation 18 as:

$$P(t) = \sum_i P(t|\sigma_i)P(\sigma_i) \quad (19)$$

where the probability of time to rupture given a certain stress, $P(t|\sigma_i)$ is known from the 3-parameter Weibull fits in the LMP analysis of Section 2.1.2, and the probability of stress $P(\sigma_i)$ is known from the Gaussian distribution fit obtained in Section 3.1. The 3-parameter Weibull distribution of the LMP is also the distribution of the time to rupture if we set the temperature in Equation 1 to 600°C and we solve for time, such that $\log(t) = 5/3LMP - 33.1$.

Performing the summation of Equation 19 using both probability distributions over the entire stress range yields the probability distribution function (PDF) for the time to rupture. By integrating the area under the PDF, we can obtain the CDF, which allows us to quantify the POF of GR91 given a certain time to rupture. Figure 9 shows both the PDF and the CDF of the time to rupture for GR91 at 600°C, alongside a red line that highlights the POF at the Section III, Division 5 primary load design life of this FHV. This cumulative probability

gives the probability of premature failure – the probability the vessel fails before the ASME design life.

Figure 9. PDF (left) and CDF (right) of the time to rupture for GR91 at 600°C. The red line shows the design life of the FHV.

For other nuclear components that are evaluated from a probabilistic perspective, such as graphite and silicon carbide components, the ASME BPVC rules give the engineer a target POF to reach in order to ensure the safety of the designed component. For components requiring the highest degree of safety, the ASME rules prescribe a target $\text{POF} \leq 1 \times 10^{-4}$, which is highly conservative.

For the current GR91 FHV, which was designed via traditional ASME primary load design rules, the calculated POF is 2.03×10^{-6} . We can therefore infer that the current ASME rules generate exceptionally conservative metallic nuclear components, which already was the general assumption of the nuclear community. Indirectly, this quantification of the POF opens up the possibility of generating components with more relaxed design requirements that still maintain a high degree of safety.

3.3 Assessing the robustness of the ASME primary load design process

The statistical analysis of this component can be used to assess the robustness of the ASME design procedure against errors in the primary load allowable stresses. Table x summarizes the analysis. The process is to suppose that the actual allowable stresses for the sample problem temperature (600°C) were a constant factor lower than the values used here from the 2017 edition of the Code. For example, new creep data might show Grade 91 is weaker than the Larson-Miller correlation used in establishing the ASME allowable stresses or the allowable stress values were simply misprinted. The component design and loading remain the same and the designer thinks the vessel has the design life shown in the second column of the table. In actuality, the design life with the correct allowable stress would be the 100,000 hour life used in the reference calculation described previously.

The statistical analysis remains exactly the same, as the loading and specimen geometry remain fixed. The probability distributions shown in Figure 9 can be used to compute the

probability the now vessel fails before the maximum design life produced from the incorrect allowable stress values. Producing this table required extrapolating the ASME allowable stress values beyond 300,000 hours, which is somewhat questionable. However, it

Table 3.1. Premature failure probabilities for designs to incorrect allowable stresses.

$S_{mt}^{correct} / S_{mt}^{incorrect}$	Incorrect design life (hr)	Correct design life (hr)	Probability of premature failure
1.0	100,000	100,000	2.0×10^{-6}
0.9	268,788	100,000	1.5×10^{-4}
0.8	806,009	100,000	4.4×10^{-3}
0.7	2,769,267	100,000	2.2×10^{-2}

The ASME design process is extremely robust. Even if the allowable stresses were wrong by 43% ($\frac{1-0.7}{0.7}$) the resulting design would still only have a 2.2% chance of premature failure. Smaller errors in the allowable stress produce very low probabilities of premature failure.

4 Conclusions and Future Work

Starting with experimental data of the material properties of GR91, we have developed an initial method for evaluating the rupture life of a component, the FHV that directly considers the variability of the material properties in its calculations. In this work, we have arrived to the following conclusions:

1. Stokes flow with non-Newtonian viscosity is an adequate representation of the creep rates and stress distribution in an equivalent steady-state elastic power-law creep model.
2. With the aid of several robust statistical methods (GP fitting, Monte Carlo, Weibull) we were able to properly obtain a probabilistic model for rupture that accounts for material variability.
3. A probabilistic approach can give us a more flexible and accurate assessment of the rupture life in a component.

To have a more complete assessment of a component's performance, we are interested to work next on including the variability of the load into this life prediction method. The resulting procedure can then be used to generate a comprehensive model for premature structural failure. Additionally, we may be also interested in utilizing this probabilistic method to inform and improve the ASME design rules so that the variability of both the material and the load are included in the design process, such that ultimately a design margin can be calculated and used by the engineer to make more informed decisions.

Additionally, this probabilistic evaluation method could inform future deterministic design methods calibrated to meet a target reliability. Load and Resistance Factor design is an example of a deterministic approach calibrated to an underlying statistical reliability. One challenge in applying this concept to high temperature, steady state, creep design is that the creep problem involves three probability distributions: load, deformation, and resistance. The classical LRFD design only involves only distributions of the load and resistance.

The statistical model developed here is only an approximation of the full problem of design against creep rupture at fixed load:

- The current model uses the stress distribution using the minimum creep rate. This neglects the time required to attain this steady state stress distribution. This is a classical approximation, also used in the ASME primary load design method, but still neglects the damage accumulated during transient creep.
- This model assumes a von Mises effective stress relating the multiaxial stress to the uniaxial rupture data. More sophisticated deterministic models better represent the multiaxial rupture data, both for GR91 and other materials. Statistical methods could then also be applied to this data.

- A complete statistical model covering all of the ASME primary load design criteria would also need to consider, in addition to the creep rupture criterion addressed here:
 - Plastic collapse
 - Strain accumulation
 - The onset of tertiary creep

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