PASSIVELY SAFE REACTOR STABILITY UNDER POST LOSS-OF-FLOW WITHOUT SCRAM CONDITIONS

by

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\( \lambda_w \) fraction of full coolant flow 
\( (= v/v_{100}) \), dimensionless

\( \lambda_{w,nc} \) fraction of full coolant flow at which the transition to natural circulation occurs, dimensionless

\( \Lambda \) mean prompt neutron generation time, s

\( \xi \) nondimensional axial coordinate 
\( (= z/H_c) \)

\( \rho \) reactivity, dimensionless

\( \tau \) coolant transport time through the core, s

\( \tau_a \) time argument, s

\( \tau_{100} \) full flow coolant transport time through the core, s

\( \phi \) nondimensional fission power amplitude (normalized to initial fission power)

\( \psi_{gk} \) nondimensional product of precursor concentration and energy release for decay heat group \( g \) and decay heat characteristic \( k \)

\( \psi_f \) nondimensional fission power amplitude normalized to initial total power

\( \psi_h \) nondimensional decay power amplitude normalized to initial total power

\( \psi_t \) nondimensional total reactor power amplitude normalized to initial total power

\( \omega \) angular frequency, rad/s

\( \omega_u \) angular frequency of oscillation at the stability boundary, rad/s

Subscripts

\( ss \) steady-state value

\( zi \) denotes the zero input portion of a response, that is, that due to initial conditions only

Superscripts

\( . \) time derivative

\( , \) deviations with respect to steady state
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ABSTRACT

Part of the reactor design process is performance evaluation according to predefined criteria including reactor stability behavior under different conditions. This work focuses on the stability characterization of a reactor system with feedback under low reactor power, low reactor coolant flow conditions. Such conditions might be encountered, for example, after a loss-of-flow without scram in some passively safe reactor designs. Algebraic and frequency stability criteria based methods are developed to find stability regions, stability boundary surface in system parameter space, and frequency of oscillation at oscillatory instability boundaries. Models are developed for the reactor, its detailed thermal hydraulic reactivity feedback path associated with coolant outlet temperature, and decay heat. Developed stability analysis tools are applied to the system model. A unique aspect is the assessment of the influence of decay heat on stability. Other selected parameters are: temperature coefficient of reactivity reactor coolant flow, and natural circulation flow. The result is a stability boundary surface in four-dimensional system parameter space and its associated frequency of oscillation surface. Adopting model parameter values from two reactors results in system parameters within the stable region. Conditions for system parameters to remain in the stable region are identified.
I. INTRODUCTION

The subject of nuclear reactor stability has been the object of many studies. Early work is reported in conferences and papers.\textsuperscript{1-5} For example, the analysis of the stability of particular reactors is given in Refs. 6 and 7. Some of the methods used in linear and nonlinear system stability are presented in Ref. 8. Work in the area of reactor stability continues; more recent work includes boiling water reactor limit cycles and nonlinear stability analysis among other areas.\textsuperscript{9-12}

As an integral part of a reactor design process, it is necessary to evaluate the design's performance with respect to different predefined criteria. One of these criteria is related to the reactor stability behavior under different conditions. Previous work includes the study of reactor stability using the feedback transfer function and use of separate high- and low-frequency conditions.\textsuperscript{11} The possibility of appearance of oscillations in the reactor power and reactivity feedback, especially under a certain set of low flow and low reactor power conditions is of interest.

Specifically, the purpose of this work is the stability characterization of a reactor system with feedback determined by the reactor outlet coolant temperature under low reactor power, low reactor coolant flow conditions. This characterization is to establish whether the system is stable or unstable for different values of system parameters, whether instabilities or oscillations are possible and under what conditions such behavior may be expected to appear, what is the margin to the stability boundary, what system parameters affect stability and how to modify them to retain or enhance stability.

Although a general stability analysis method is developed and used, the purpose is to characterize the reactor system behavior primarily under specific system conditions, namely, low reactor power, low reactor coolant flow. Such conditions may be encountered, for example, in a post loss-of-flow without scram situation of some passively safe reactor designs where the reactor is designed to cut back power production and eventually go to and remain in a low-power, low-flow state in which all power produced is removed by passive means for an
indefinite period of time, and where all events in this sequence occur without requiring human or active system intervention.

The influence of decay heat on stability, which does not appear to be extensively reported in the literature, is a novel concept addressed also in this work. For any event sequence such as the above that leads to a low-flow low-power condition the total power is reduced significantly from its full power level. The decay power which was at most a small fraction of total power under full power conditions assumes a larger relative importance under low-power conditions. This, in turn, has an effect on the stability behavior of the reactor system.

In a reactor, many reactivity feedback effects are simultaneously present; all are not considered here, only one reactivity feedback path is the object of interest: the feedback component associated with the reactor outlet coolant temperature. In steady oscillations, at constant coolant flow rate, the reactor coolant temperature at any axial position lags the reactor power, and the reactor outlet coolant temperature is associated with the largest lag with respect to reactor power. Thus, by selecting the reactor outlet coolant temperature, in a sense, a bounding stability analysis is performed.

The analysis of reactor stability behavior is carried out in the linear systems theory framework. The analysis method is general in character. The expected results of such stability analysis may be presented as a stability map in the system parameter space, showing the stable and unstable regions and the stability boundary surface. In the cases where an oscillatory instability boundary exists, further characterization is given by the frequency of oscillation on the stability boundary surface points.

The body of this work consists of essentially three parts. The first part presents the methodology to characterize the stability behavior of linear systems for all combinations of system parameters. One of the algebraic stability criteria is selected and the procedure for the determination of the stability boundary surface is described. Another method is used to determine the frequency of oscillation for oscillatory instability boundaries. The second part
models the physical phenomenon under study. Models of the reactor, of the detailed thermal hydraulic feedback, including the effects of coolant transport and heat conduction, and of the decay heat are developed. The third part consists of the application of the tools developed in the first part to the physical system modeled in the second part. The results characterizing the stability of the system for all combinations of selected system parameters are shown. Data obtained from the literature for two reactors are adopted for a model parameter. The resulting calculations illustrate the stability prediction procedure and the conditions under which system parameters move beyond or remain within the stable region. Finally, results are summarized and conclusions are presented.

II. REACTOR STABILITY CHARACTERIZATION

A. Stability Boundary Surface Determination

In order to assess system stability for different system parameters, Routh’s algebraic stability criterion\(^\text{13}\) is selected. The method developed for this work is a two-step process. The first step consists of sweeping the parameter space and applying the Routh criterion at each point of the space to determine the number of roots in the right half-plane. This procedure yields all the $D(m)$ regions, where $D(m)$ is the region associated with $m$ right half-plane roots, and, in particular, the sought-for stability region $D(0)$. This first step generates a stability map with a resolution as fine as the one used in the sweeping of the parameter space which is limited in practice. To determine the precise location of the boundary of stability, a different technique is used in the second step: a line search of the boundary is performed by holding constant all but one of the system parameters. The boundary is detected by the discontinuity in the number of right half-plane roots given by the application of the Routh criterion. The line search is repeated for all values of the system parameters initially held constant. This procedure yields the precise location of the stability boundary surface in system parameter space.
B. **Frequency of Oscillation Determination**

In the cases where an oscillatory instability develops when system parameters cross over the stability boundary, further characterization of the instability phenomenon is provided by the oscillation frequency at the stability boundary crossover point. To derive the conditions that determine the oscillation frequency, a root locus\(^{14}\) type of analysis is developed. This analysis yields an equation for the oscillation frequency \(\omega_\alpha\), and an expression for the gain \(\lambda_\alpha\) at the crossover point. The following sections describe the model that was adopted to represent a reactor and its feedback mechanism. The stability of this system is analyzed using the method presented in this and in the previous section. The reactor and its feedback mechanism along with decay heat effects are incorporated in the model.

III. REACTOR MODEL

For a given reactor, the objective is to determine the stability boundary conditions using an appropriate model. The reactor is represented using a point kinetics model with all precursor groups. The initial nonlinear model is (see Nomenclature):

\[
\dot{\phi} = \frac{\rho - \beta}{\Lambda} \phi + \sum_{i=1}^{n_r} \lambda_i c_i, \tag{1}
\]

\[
\dot{c}_i = \frac{\beta_i}{\Lambda} \phi - \lambda_i c_i. \tag{2}
\]

Linearizing with respect to the steady state associated with \(\phi = \phi_s\), and taking Laplace transforms yields, after some manipulation, the transfer function between the reactivity changes and fission power amplitude changes normalized to an initial steady-state level:
\[
\frac{\phi'(s)/\phi_{ss}}{\rho'(s)} = \frac{1}{s \left[ \Lambda + \sum_{i=1}^{n} \frac{\beta_i}{s + \lambda_i} \right]},
\]

where the prime indicates deviations from the steady state. Define:

\[
G_i(s) = \frac{1}{s \left[ \Lambda + \sum_{i=1}^{n} \frac{\beta_i}{s + \lambda_i} \right]}.
\]

If the fission power amplitude is normalized to the steady-state value, then \( \phi_{ss} = 1 \), and:

\[
\frac{\phi'(s)}{\rho'(s)} = G_i(s).
\]

IV. DECAY HEAT MODEL

As mentioned in the introduction, for a reactor under low-flow, low-power conditions, it is of interest to consider the influence of decay heat in the stability analysis. A model with several sets of different decay heat characteristics similar to the one in Ref. 15 is used:

\[
\dot{\psi}_{gk} = \beta_{gk} \Psi_f(t) - \lambda_{gk} \psi_{gk};
\]

where \( g \) is the decay heat group and \( k \) denotes the decay heat characteristic. The total decay power is a weighted sum of over all decay heat characteristics and all decay heat groups used:
\[ \Psi_h(t) = \sum_k w_k \sum_g \lambda_{hgk} \psi_g(t) . \] (7)

Writing Eqs. (6) and (7) in terms of deviations from the steady state associated with \( \Psi_f = \Psi_{fas} \), taking Laplace transforms, and retaining for generality the initial \( (t = 0^+) \) state quantities, yields, after some manipulation, the following expression written as the sum of the zero state and the zero input responses:

\[ \Psi'_h(s) = G_h(s) \Psi'_f(s) + \Psi'_{h,zi}(s) ; \] (8)

where two quantities have been defined: (1) the transfer function between the fission power amplitude changes and the decay heat amplitude changes, each normalized to the initial total power level:

\[ G_h(s) = \sum_k w_k \sum_g \frac{\beta_{hgk} \lambda_{hgk}}{s + \lambda_{hgk}} ; \] (9)

and (2) the zero input portion of the response:

\[ \Psi'_{h,zi}(s) = \sum_k w_k \sum_g \frac{\lambda_{hgk}}{s + \lambda_{hgk}} \psi'_g(0^+) . \] (10)

V. THERMAL HYDRAULIC FEEDBACK MODEL

As the power varies, temperatures change and reactivity feedback effects appear. The stability analysis utilizes the transfer function describing the feedback effects. The determination of the feedback transfer function from power to reactivity requires an understanding of the
temperature effects. A key link is the way reactor power affects coolant temperature. The behavior of coolant temperature is the focus of the following section.

A. Coolant Temperature Behavior

The determination of coolant temperature for constant coolant velocity is addressed in this section. Axially continuous coolant temperature and radially continuous fuel temperature distributions are assumed. Other assumptions include: no axial heat conduction in either fuel or coolant; no axial mixing in the coolant; all material properties (density, specific heat, thermal conductivity), heat transfer coefficients, and velocity are independent of temperature; and no heat generation in the coolant. The development follows the so-called "exact" model in Ref. 16.

The coolant temperature \( T_c(z,t) \), at an axial position \( z \) and time \( t \), for a given coolant velocity \( v \), is given by:

\[
C'_e \left( \frac{\partial T_c(z,t)}{\partial t} + v \frac{\partial T_c(z,t)}{\partial z} \right) = q'_F(z,t). \tag{11}
\]

At any axial position, the linear heat flow rate from fuel to coolant, \( q'_F \), is entirely determined by the past history of coolant temperature and generated power. For a steady oscillation, this means that \( q'_F \) can be expressed as a function of \( T_c \) and \( q' \). It can be shown that this function may be represented by the form:

\[
q'_F(z,s) = A(s)T_c(z,s) + B(s)q'(z,s); \tag{12}
\]

where \( A(s) \) and \( B(s) \) are some appropriate transfer functions, and are not a function of \( z \). Equation (11) can then be solved for the coolant temperature to yield the following transfer function from power to coolant temperature:
\[ R(\xi, s) = \hat{T}_{10} P_2(s) r e^{(s-P_2(s))t} \int_0^\xi e^{(s-P_2(s))\xi'} q'(\xi') d\xi'; \]  

(13)

where:

\[ P_1(s) = \frac{A(s)}{C_c'}; \]  

(14)

and

\[ P_2(s) = \frac{C_c'}{C_c'} B(s). \]  

(15)

\[ \text{B. Heat Flow Rate from Fuel to Coolant} \]

The purpose of this section is to find the linear heat flow rate from fuel to coolant \( q'(z,s) \) for a cylindrical fuel element, assuming material properties and power density are uniform in the fuel element cross section. For simplicity, the variable \( z \) is dropped from the notation. The fuel temperature \( T_f(r, t) \) at radial coordinate \( r \) and time \( t \), satisfies the time dependent heat conduction equation:

\[ \frac{1}{\kappa_f} \frac{\partial T_f(r, t)}{\partial t} = \frac{q'(t)}{\pi r_f^2 \lambda_f} + \nabla^2 T_f(r, t). \]  

(16)

The boundary condition is:
\[ -\lambda_f \left[ \frac{\partial T_f(r,t)}{\partial r} \right]_{r=r_f} = \alpha_f (T_f(r_f,t) - T_e(t)). \]  

(17)

Equations (16) and (17) can be solved for the fuel temperature to derive the heat flow rate from fuel to coolant:

\[ q'_f(s) = \frac{2}{b J_0(b)} \left( \frac{\pi \lambda_f b^2 T_c(s) + q'(s)}{\lambda_f b^2} \right). \]  

(18)

Restoring the variable \( z \) and comparing with Eq. (12) gives:

\[ A(s) = \frac{-2\pi \lambda_f \frac{s}{\kappa_f} r_f^2}{\sqrt{-\frac{s}{\kappa_f}} r_f J_0 \left( \sqrt{-\frac{s}{\kappa_f}} r_f \right) \left( \lambda_f \frac{s}{\kappa_f} r_f + \frac{\lambda_f}{\alpha_f} \frac{s}{\kappa_f} r_f \right) \sqrt{J_1 \left( \sqrt{-\frac{s}{\kappa_f}} r_f \right)}}; \]  

(19)

\[ B(s) = \frac{2}{\sqrt{-\frac{s}{\kappa_f}} r_f J_0 \left( \sqrt{-\frac{s}{\kappa_f}} r_f \right) \left( \lambda_f \frac{s}{\kappa_f} r_f + \frac{\lambda_f}{\alpha_f} \frac{s}{\kappa_f} r_f \right) \sqrt{J_1 \left( \sqrt{-\frac{s}{\kappa_f}} r_f \right)}}; \]  

(20)

and:

\[ A(s) = -s C'_f B(s). \]  

(21)
Using Eqs. (14), (15) and (21) gives:

\[ P_2(s) = -\frac{P_1(s)}{s}. \]  

(22)

C. **Thermal Hydraulic Transfer Function**

If the oscillations that take place are of a relatively low frequency, an approximate simplified expression referred to as an equilibrium heat exchange approximation may be derived. Using Eq. (20), it can be shown that as \( s = j\omega \to 0 \), \( B \) approaches unity:

\[ \lim_{s \to 0} B(s) = 1 + 0(s). \]  

(23)

Using Eqs. (14) and (21) yields:

\[ P_1(s) = -\frac{sC'_f}{C'_c} B(s). \]  

(24)

Thus, for low frequencies, a good approximation is:

\[ P_1(s) = -\frac{sC'_f}{C'_c} = -\epsilon s; \]  

(25)

and, using Eq. (22):

\[ P_2(s) = -\frac{P_1(s)}{s} = \epsilon. \]  

(26)
Substitution of Eqs. (25) and (26) into Eq. (13) yields:

\[ R(\xi, s) = (\dot{\bar{T}}_{10}/\bar{\varepsilon}) \ e^{-((1 + \varepsilon) s)\tau} \int_0^\xi e^{((1 + \varepsilon) s)\tau'} q'_t(\xi') d\xi'. \]  

(27)

To obtain the transfer function between reactor power and outlet coolant temperature, Eq. (27) is evaluated at \( \xi = 1 \) assuming an axially uniform power profile. The result is:

\[ R(\xi = 1, s) = (\dot{\bar{T}}_{10}/\bar{\varepsilon}) \ \frac{1 - e^{-(1 + \varepsilon) s\tau}}{(1 + \varepsilon) s\tau}; \]  

(28)

where the first expression in parenthesis on the right side is simply the coolant temperature rise across the core.

The transfer function between reactor power amplitude changes and outlet coolant temperature changes, under the equilibrium heat exchange approximation is given by Eq. (28) which with a slight change in notation becomes:

\[ \frac{T'_{co}(s)}{\Psi'_t(s)} = \Delta T_c \ \frac{1 - D_\infty(s, (1 + \varepsilon) \tau)}{(1 + \varepsilon) \tau \ s}; \]  

(29)

where \( D_\infty \) denotes the function:

\[ D_\infty(s, \tau) = e^{-s\tau}. \]  

(30)

Among other possible\(^7\) approximations, this transcendental function may be approximated as closely as desired by choosing an appropriate integer \( n_a \) in the rational function:
\[
D_{n_s}(s, \tau_s) = \left( \frac{1 - \frac{\tau_s s}{2 n_s}}{1 + \frac{\tau_s s}{2 n_s}} \right)^{n_s}.
\]

The representation of \(D_\infty(s, (1 + \epsilon)\tau)\) as a ratio of high-order polynomials is accurate within a prescribed maximum error of less than a fraction of a percent. The coolant transit time through the core \(\tau\) may be expressed in terms of the its value at full coolant flow, and the fraction of full coolant flow \(\lambda_w\) as:

\[
\tau = \frac{\tau_{100}}{\lambda_w}.
\]

The stability analysis is carried out for different coolant flow values. It is assumed that the flow is set and maintained at a some constant value and that the coolant temperature rise across the core \(\Delta T_c\) is constant. With an appropriate experimental setup, flow can be made as low as desired. However, in a real reactor, flow is controlled by the reactor coolant pumps; when these are turned off, flow goes to a nonzero natural circulation level. This transition flow level is dependent on the reactor and heat transport system thermal hydraulic design. At and below the transition flow, designated as \(\lambda_{w,\infty}\), the power-flow relation is governed by the laws of natural circulation. The relevant fact is that \(\Delta T_c\) decreases with decreasing flow. The specific form of this function depends on design parameters and the flow regime. An approximate relation is:

\[
\Delta T_c \propto \lambda_w^{2-n_r};
\]

where \(n_r\) is flow regime dependent. For highly turbulent flow, \(n_r = 0.2\); and for laminar flow, \(n_r = 1\). Equation (33) is obtained using the fact that the natural circulation flow is proportional to the power 1 / (3 - \(n_r\)) of the reactor thermal output, and that the coolant temperature rise
across the core is proportional to the ratio of reactor thermal output and coolant mass flow rate. In this analysis, a linear decrease in core temperature rise for flows below the transition flow is assumed:

$$\Delta T_c = \min \left[ 1, \frac{\lambda_w}{\lambda_{w,nc}} \right] \Delta T_{c,100}. \quad (34)$$

Since in the turbulent regime the relationship in Eq. (33) is almost quadratic this assumption is conservative with respect to stability analysis. Use of Eqs. (31), (32) and (34) into Eq. (29) results in:

$$\frac{T_{\infty}'(s)}{\Psi'(s)} = H_{th}(s, \lambda_w, \lambda_{w,nc}); \quad (35)$$

where:

$$H_{th}(s, \lambda_w, \lambda_{w,nc}) = \min \left[ 1, \frac{\lambda_w}{\lambda_{w,nc}} \right] \Delta T_{c,100} \frac{1 - D_{ns} \left( s, \frac{(1 + \epsilon) \frac{\tau_{100}}{\lambda_w}}{(1 + \epsilon) \frac{\tau_{100}}{\lambda_w}} \right) \Delta T_{c,100}}{(1 + \epsilon) \frac{\tau_{100}}{\lambda_w}}. \quad (36)$$

D. Power Amplitudes

The total power is the sum of fission and decay power contributions:

$$P(t) = P_f(t) + P_n(t); \quad (37)$$

which written in terms of deviations from the steady state becomes:
\[ \Psi'_t(t) = \Psi_f(0)\phi'(t) + \Psi'_h(t) - (\Psi_h(0) - \Psi_{h,at}); \]  
(38)

since the analysis starts at a initial point of steady state for fission power but allows for a decay heat level that may be different from that one corresponding to the existing steady-state fission power level. Application of the Laplace transform to Eq. (38) and substitution of Eq. (8) yields:

\[ \Psi'_t(s) = \Psi_f(0)\phi'(s) + G_h(s)\Psi'_f(s) + \left[ \Psi'_h,zi(s) - \frac{(\Psi_h(0) - \Psi_{h,at})}{s} \right]. \]  
(39)

For the purposes of stability analysis, the frequencies of interest are such that the effects of decay heat are very delayed; therefore, the last two terms in Eq. (39) may be neglected implying that, at the frequencies of interest, both the decay heat contribution originating from fissions occurring after \( t = 0 \) is very small compared with the fission power component, and that the decay heat level originating from fissions occurring before \( t = 0 \) is essentially constant, respectively. With these conditions, Eq. (39) becomes:

\[ \Psi'_f(s) = \Psi_f(0)\phi'(s); \]  
(40)

or:

\[ \Psi'_t(s) = \left[ 1 - \frac{\lambda_h}{\lambda_p} \right] \phi'(s); \]  
(41)

where the following definitions have been used:

\[ \lambda_h = \frac{P_h(0)}{P_{100}}, \]  
(42)
\[ \lambda_p = \frac{P_i(0)}{P_{100}}. \] (43)

Even though dynamic effects of decay heat have been neglected, its assumed constant value still enters into the system model through the \( \lambda_h \) parameter in Eq. (41). The stability analysis is performed assuming initial fractional power and fractional flow are the same:

\[ \lambda_p = \lambda_w. \] (44)

This is equivalent to assuming some constant coolant temperature rise across the core for all flows. This or other similar condition provides a consistent basis for comparison of the stability analysis results at different coolant flows. Equation (41) then becomes:

\[ \Psi'(s) = \left(1 - \frac{\lambda_h}{\lambda_w}\right) \phi'(s). \] (45)

Strictly, Eq. (44) is valid for \( \lambda_w \geq \lambda_{w,nc} \). Below \( \lambda_{w,nc} \), since \( \Delta T_c \) decreases with decreasing flow, \( \lambda_p \), being proportional to the product of coolant temperature rise and coolant flow, decreases faster than \( \lambda_w \). Thus, Eq. (44) is an upper bound for \( \lambda_p \) and its use in Eq. (41) is conservative with respect to stability analysis.

E. Reactivity Feedback

The reactivity feedback is assumed proportional to the outlet coolant temperature. In terms of deviations with respect to steady state:

\[ \rho'(t) = k T'_c(t) = \lambda_k k_0 T'_c(t); \] (46)

where the temperature coefficient of reactivity \( k \) is expressed as the product of a base value \( k_0 \) and a multiplier \( \lambda_k \).
VI. RESULTS

The models in previous sections constitute building blocks to represent the reactor system with feedback under study. A system model may be constructed: it includes Eqs. (4), (5), (45), (36), (35), and (46). The fission power amplitude \( \phi(t) \) is normalized to the initial fission power. Since the initial state is a steady state for fission power, Eq. (5) may be used. The diagram of this system is shown in Fig. 1. Figure 1 also indicates the presence of system parameters \( \lambda_k, \lambda_h, \lambda_w \) and \( \lambda_{w,nc} \) used in the stability analysis.

![Block Diagram for a Reactor System With Feedback](image)

Fig. 1. Block Diagram for a Reactor System With Feedback

The focus of this work is the characterization of the stability behavior of the reactor system under study as the four parameters \( \lambda_k, \lambda_h, \lambda_w, \) and \( \lambda_{w,nc} \) span the system parameter space. The stability analysis is performed for different coolant flow rates. For each coolant flow rate level it is assumed that the initial conditions are the following. The fission power has reached some steady-state level after all internal and external feedback mechanisms have actuated and settled, and all thermal power produced is removed by the coolant flow. For coolant flows above the natural circulation transition flow, the total power level expressed as a fraction of full power is the same as the coolant flow rate fraction of full flow (\( \lambda_p = \lambda_w \)); the coolant temperature rise across the core is unchanged from its full power value. For coolant flows
below this natural circulation transition flow, the coolant temperature rise across the core decreases with decreasing coolant flow and, at a faster rate, so does power. The decay heat level is determined by previous history and is independent of coolant flow level. The model parameters for the reactor are those of a large liquid-metal reactor (LMR). Other selected parameters are given in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_c$</td>
<td>0.9652</td>
<td>m</td>
</tr>
<tr>
<td>$k_0$</td>
<td>-10^{-6}</td>
<td>K^{-1}</td>
</tr>
<tr>
<td>$v_{100}$</td>
<td>6.7</td>
<td>m s^{-1}</td>
</tr>
<tr>
<td>$\Delta T_{c,100}$</td>
<td>153</td>
<td>K</td>
</tr>
<tr>
<td>$e$</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$\tau_{100}$</td>
<td>0.1441</td>
<td>s</td>
</tr>
</tbody>
</table>

The results of the calculation of the stability boundary surface are shown in Fig. 2. The plot shows the value $\lambda_k$ that puts the system on the limit of the stability region as a function of the coolant flow rate level $\lambda_w$, parametrically in the decay heat level $\lambda_h$. The natural circulation transition flow $\lambda_{w,nc}$ is taken as zero for this calculation. The region below any particular curve gives the set of all the combinations of system parameters that result in a stable system; the region above corresponds to all unstable systems. For each decay heat level, no curve is shown for $\lambda_w < \lambda_k$ since the coolant flow must be large enough to remove at least the decay heat; otherwise, no initial steady state is possible.

Calculations performed using this model up to full flow (not shown) indicate that the stability boundary $\lambda_k$ values increase monotonically with flow $\lambda_w$, after possibly reaching a minimum, following the trend observed in Fig. 2. This may be expected since as $\lambda_w$ increases, the coolant transit time based delay decreases and increased amplification of $k_0$ is required to
reach the stability boundary. The range of interest is primarily that of lower flows. At higher flows, the conditions for stability become less restrictive, and the implicit equilibrium heat exchange approximation becomes less accurate.

Figure 2 shows, for the case of no decay heat ($\lambda_h = 0$), that as the coolant flow decreases, so does $\lambda_k$. In other words, as transit time based delay increases, the maximum value of the temperature coefficient of reactivity $k$ that can be tolerated while still remaining in the stable region is progressively reduced. The presence of decay heat results in a less restrictive condition on $\lambda_k$, the stability boundary surface reveals this progressively less restrictive condition for progressively higher values of the decay heat, as seen in Fig. 2. The stabilizing effect of
decay heat on the reactor behavior may be explained as follows. In the case of no initial decay heat, all reactor power is determined by reactivity, which is controlled by outlet coolant temperature, which, in turn, is related to power; thus closing the feedback loop (Fig. 1). In the case where decay heat is present, a fraction of the total reactor power is of decay heat origin, and its initial value is determined by the power history. Therefore, the fraction of total reactor power that is directly controlled by reactivity, through outlet coolant temperature, is reduced; and the feedback loop is weakened since the influence of the perturbations of temperature on total reactor power level through reactivity is diminished for any fixed \( \lambda_k \). Thus, for a weakened loop, the stability boundary occurs for higher \( \lambda_k \). In the limit \( \lambda_w \to \lambda_h \), the loop tends to break and \( \lambda_k \to \infty \). These singular points may be observed in Fig. 2. Reactivity does influence the decay power, but only indirectly through changes in the power history, thus providing only a weaker, delayed link; for this reason this link was not retained in the model.

The nature of the thermal hydraulic feedback suggests the possibility of oscillatory behavior and therefore of an oscillatory instability boundary. This is confirmed by the fact that when crossing over the stability boundary, the number of right half-plane roots of the characteristic equation increases by two indicating the appearance of a pair of (complex conjugate) roots.

Using the methods mentioned previously, the frequency of oscillation at the stability boundary is computed. The result is shown in Fig. 3. The plot gives the frequency of oscillation \( \omega_u \) on the limit of the stability region as a function of the coolant flow rate level \( \lambda_w \), for all decay heat levels, and for all natural circulation transition flow levels. The influence of changes in the values of the decay heat and of \( \lambda_{w,nc} \) may be incorporated into an overall root locus gain. Since the equation that determines the frequency of oscillation is independent of this gain, so is the oscillation frequency. Thus, all curves parametric in \( \lambda_h \) and in \( \lambda_{w,nc} \) coincide. Figure 3 is applicable when the stability boundary is defined, as shown in Fig. 2 \( (\lambda_w > \lambda_h) \).

Figure 3 shows the frequency of oscillation increasing monotonically with the coolant flow fraction. This is to be expected since the oscillations are made possible in part by the delay associated essentially with the coolant transit time through the core. The higher the coolant flow
rate, the lower the transit time and the oscillation period, and consequently the higher the oscillation frequency.

![Graph showing the relationship between \( \lambda_w \) and \( \omega_u \)](image)

**Fig. 3.** Frequency of Oscillation on the Stability Boundary Surface for a Reactor System With Feedback for All Decay Heat Levels and All Natural Circulation Transition Flow Levels

The oscillation period at the stability boundary is driven by, but is not exactly equal to the coolant transit time. From Fig. 3, very approximately \( \omega_u \approx 10 \lambda_w \text{ rad s}^{-1} \), so \( T_u = \frac{2\pi}{10\lambda_w \text{ rad s}^{-1}} = \frac{\pi s}{(5\tau_{100})} \approx 4.4\tau \). The bulk of the period amplification (a factor of 4 in the Fig. 3 case) relative to the coolant transit time is mainly associated with the detailed dynamics of the heat transfer processes on the thermal hydraulic feedback path. The rest is related to the reactor dynamics.

The results of the calculation of the stability boundary surface are shown in Fig. 4 as a function of the coolant flow rate level \( \lambda_w \), for a zero decay heat level \( \lambda_0 = 0 \), parametrically in
in the natural circulation transition flow $\lambda_{w,nc}$. The zero decay heat is selected since for a stable system it provides the most restrictive stability boundary surface of all decay heat levels. For high enough coolant flows, all curves merge into one. As the flow decreases, each parametric curve branches out from this one at the transition flow that corresponds to the point of maximum power that can be removed by natural circulation. Further decreases in coolant flow correspond to points of lower power and decreasing coolant temperature rise across the core within the natural circulation regime. The higher the natural circulation transition flow, for any fixed coolant flow $\lambda_w < \lambda_{w,nc}$, the lower the coolant temperature rise across the core, and thus the influence of power changes on reactivity. The feedback loop (Fig. 1) is weakened, and increasingly higher values of $\lambda_k$ are required to put the system on the stability boundary, as may be noted in Fig. 4.

![Graph showing stability boundary surface for a reactor system with feedback for a zero decay heat level and different natural circulation transition flow levels $\lambda_{w,nc}$](image-url)

**Fig. 4.** Stability Boundary Surface for a Reactor System With Feedback for a Zero Decay Heat Level and Different Natural Circulation Transition Flow Levels $\lambda_{w,nc}$. 

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Figure 5 shows the stability boundary surface, computed for the least favorable case of no stabilizing decay heat, along with the values derived from the literature of the parameter \( \lambda_k \) for two reactors\(^{19,20} \). In the case of an actual calculation for a particular reactor design, the design parameters must be compared with the stability boundary determined for that particular design.

![Computed Stability Boundary Surface](image)

**Fig. 5.** Computed Stability Boundary Surface for a Reactor System With Feedback and \( \lambda_k \) Values from the Literature (\( \lambda_h = 0, \lambda_{w,nc} = 0.05 \))

The literature \( \lambda_k \) values are listed in Table II. They are determined by the temperature coefficient of reactivity associated with the outlet coolant temperature in what is referred to as Type A cases in Table II. The Type B case values in Table II are determined by the total temperature coefficient of reactivity. Using this value for \( \lambda_k \) is equivalent to assuming that all feedback effects are concentrated in the outlet coolant temperature feedback path. A single base value \( k_0 \), given in Table I, is used throughout this work. Some entries in the Type column of Table II also indicate the type of reactor fuel.
TABLE II. $\lambda_k$ Parameter Values

<table>
<thead>
<tr>
<th>Case</th>
<th>Reference</th>
<th>Type</th>
<th>$\lambda_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>A</td>
<td>5.3640</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>B</td>
<td>55.080</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>A-U</td>
<td>1.3700</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>B-U</td>
<td>34.000</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>A-Pu</td>
<td>1.2600</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>B-Pu</td>
<td>28.900</td>
</tr>
</tbody>
</table>

Specifically, in Table II the Case 1 $\lambda_k$ value corresponds to the sum of coolant density, coolant displacement, steel density, and $\text{B}_4\text{C}$-fuel components of the temperature coefficient of reactivity in the above-core region for the Experimental Breeder Reactor II (EBR-II) Run 122A core loading (Ref. 19, Table 6). The Case 2 $\lambda_k$ value is associated with the sum of coolant density, coolant displacement, steel density, bond sodium, fuel axial expansion, Doppler, and $\text{B}_4\text{C}$-fuel components of the temperature coefficient of reactivity in all reactor regions, plus the sum of net rod-bank suspension and grid-plate radial-expansion effects associated with a coolant temperature change at pool inlet but assuming fixed tank wall temperature, for EBR-II Run 122A core loading (Ref. 19, Table 6). The Case 3 value corresponds to the sum of coolant density, coolant displacement, and steel density components of the temperature coefficient of reactivity in the above-core region for a U10Zr-fueled 900 MWt homogeneous LMR (Ref. 20, Table 4). The Case 4 value accounts for the sum of coolant density, coolant displacement, steel density, bond sodium, fuel axial expansion, and Doppler components of the temperature coefficient of reactivity in all reactor regions, plus the sum of net rod-bank suspension and grid-plate radial-expansion effects associated with a coolant temperature change at pool inlet but assuming a tank wall temperature change half of pool change for a U10Zr-fueled 900 MWt homogeneous LMR (Ref. 20, Table 4). Cases 5 and 6 are the same as Cases 3 and 4, respectively, except for the fact that they refer to a UPu10Zr-fueled reactor instead.
Figure 5 shows the $\lambda_k$ associated with the stability boundary surface. By comparison with the actual reactor $\lambda_k$ values, a stability prediction may be made. An assumption regarding the natural circulation transition flow value must first be made. This value is typically on the order of a few percent of full flow, the precise value being design dependent. To illustrate, a value of $\lambda_{w,nc} = 0.05$ is used. As may be observed in Fig. 5, there is an adequate margin between the actual $\lambda_k$ values for any of the two reactors and those at the stability boundary, predicting stable behavior.

If $\lambda_{w,nc}$ is sufficiently low, a situation might occur such as that illustrated in Fig. 6, constructed for $\lambda_{w,nc} = 0.005$. This figure suggests that for some cases of Table II, at sufficiently low flows, $\lambda_k$ would exceed that of the stability boundary surface therefore predicting an unstable behavior with respect to the feedback path under consideration.

![Graph showing stability boundary surface](image)

**Fig. 6.** Computed Stability Boundary Surface for a Reactor System With Feedback and $\lambda_k$ Values from the Literature ($\lambda_n = 0, \lambda_{w,nc} = 0.005$)
The constant $\lambda_k$ plots of Fig. 6 may be aggregated into two groups: Cases 1, 3 and 5 are Type A cases; Cases 2, 4 and 6 are Type B. Type A cases are physical in the sense that they assign to $\lambda_k$ a value determined by the temperature coefficient of reactivity on the outlet coolant temperature feedback path; have lower values than Type B cases; and, as seen in Fig. 6, do not violate the stability boundary condition. Thus, the modeled reactor, with the Type A $\lambda_k$ of any of the two reactors, is stable.

Type B cases are unphysical in the sense that they assign to $\lambda_k$ a value determined by the total temperature coefficient of reactivity; have higher values than Type A cases by about an order of magnitude; and, as seen in Fig. 6, do not satisfy the stability boundary condition. Thus, the modeled reactor, with the Type B $\lambda_k$ of any of the two reactors, is beyond the stable region. Since the stability condition depends on $\lambda_{w,sc}$ and $\lambda_k$, and these are design dependent parameters, this hypothetical situation may be avoided by proper design of the reactor system.

In summary, even in this low-$\lambda_{w,sc}$ case, using the actual (Type A) physical parameters of the two reactors results in a system in the stable region, separated by a margin from the stability boundary. Only in the hypothetical Type B cases, with a relatively large $\lambda_k$, and under this small natural circulation transition flow, would the system parameters go beyond the stable region. However, this situation may be prevented by adequate design of the reactor system.

VII. CONCLUSIONS

This work focuses on the stability analysis of a reactor system with feedback determined by the reactor outlet coolant temperature, under low reactor power, low reactor coolant flow conditions such as may be found, for example, in a post loss-of-flow without scram condition in some passively safe reactor designs. The novel concept of the influence of decay heat on stability is introduced and incorporated in the analysis.

The analysis is performed within the linear framework. A number of tools are developed to characterize the stability behavior of linear systems for all combinations of system parameters.
Specifically, based on the Routh criterion, a method is described to find the stability regions and the stability boundary surface in system parameter space. Based on the root locus method, a formulation is used for the determination of the frequency of oscillation for oscillatory instability boundaries.

In any given reactor there are multiple feedback paths, and its actual behavior is determined by the relative importance of different feedback mechanisms present as a result of its design. This work considers one particular case, namely, that in which the reactivity feedback is determined by the outlet coolant temperature. A model of the reactor and the thermal hydraulic feedback is developed. The reactor is represented with a point kinetics model with all precursor groups. A detailed model is used to find the reactor power to outlet coolant temperature transfer function. The model uses axially continuous coolant and radially continuous fuel temperature representations. Decay heat effects are also treated and incorporated.

The stability analysis tools are applied to the reactor with feedback model. The results show the stability boundary surface as the temperature coefficient of reactivity multiplier $\lambda_\gamma$ as a function of reactor coolant flow rate, decay heat level, and natural circulation transition flow level. Since an oscillatory instability boundary is found, the frequency of oscillation is also computed.

The results are compared with corresponding parameters of two reactors. This comparison indicates that the modeled reactor system, designed with the actual $\lambda_\gamma$ present in these reactors in the particular outlet coolant temperature feedback path under study, has its system parameters within the stable region, with a certain margin to the stability boundary. Conditions that may cause the system parameters to move beyond the stable region, and parameters that may be used to avoid this hypothetical situation by appropriate design of the reactor system are identified.
REFERENCES


